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KENNETH P. WILLIAMS

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No. 21

THE DYNAMICS OF THE AIRPLANE

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PREFACE

It was the good fortune of the author to attend the University of Paris during the spring semester of 1919. One of the special courses which the French authorities, with their characteristic hospitality, arranged for the large number of students from the American army, was a course in aërodynamics, given by Professor Marchis. The comprehensive knowledge that Professor Marchis possessed of all branches of the new science of aëronautics, the inestimable value of his advice to the French Republic during the war, the interest he took in his rather unusual class, could not fail to be an inspiration.

This book is an outgrowth of those parts of Professor Marchis' lectures that were of particular interest to the author. It is in no sense a complete treatise on aviation. Questions of design and construction are passed over with bare mention. The book is intended for students of mathematics and physics who are attracted by the dynamical aspect of aviation. The problems presented by the motion of an airplane are novel and fascinating. They vary from the most pleasing simplicity to the most stimulating difficulty. The question of stability, particularly, exhibits at the same time the elegance and the power of analysis, and shows the adaptability of some of the general developments in dynamics. The field is assuredly a fruitful one of study, and increasing demands will be put upon the mathematician as the science of aviation continues its rapid development. The mathematician can well own a sense of pride that he had already at hand, in the developments inaugurated by Euler and Routh, a means of dealing accurately with the question of stability, that plays so fundamental a rôle in the science of flying.

The treatment in the text is for the most part elementary. The last chapter alone demands of the student familiarity with more advanced dynamical methods. In the treatment of descent a slight digression is made to consider in part the nature of the solution of a system of two differential equations. This was done in order not to completely evade what seems a problem of considerable difficulty. It might seem that a treatment of the propeller should not find a place in a book with the purpose of this one. No student of mathematics, however, could fail to own a curiosity as to a propeller's action, and it is hoped the discussion, while not complete, will at least serve as a sufficient introduction.

The various curves in the text were plotted by Mr. R. W. Smith, a former student in this university. The author is further indebted to the Smithsonian Institution for permission to use Figs. 12 and 49.

In addition to the various books that are referred to in the text the author has made use of his notes of the lectures of Professor Marchis, translated into English by Madame Ciolkowska, who rendered most valuable aid as an interpreter for those who understood and spoke the language of Professor Marchis only with difficulty.

K. P. Williams.

Indiana University,
July, 1920.

THE DYNAMICS OF THE AIRPLANE

CHAPTER I

THE PLANE AND CAMBERED SURFACE

1. The possibility of aërial navigation depends upon the solution of two problems, the problem of sustentation and the problem of propulsion. At the very outset two distinct courses are therefore open. We can look upon the problems as entirely separated from each other, or we can regard them as essentially connected.

In the first case we look for separate solutions, solving first the problem of sustentation, and then, with this successfully disposed of, search for a means of propulsion. This course is historically the older and it is the simpler, for it meets the difficulties one at a time.* A balloon filled with a light

*Early literature abounds with mythical accounts of the flights of legendary heroes equipped birdlike with wings. Among those who seriously studied the question of flight, and actually designed machines with wings to be attached to a person and driven by his own muscular power, Leonardo da Vinci, the renowned artist, occupies the first place. The first instance of people actually ascending from the earth took place November 21, 1783, at Paris. The apparatus was a balloon constructed by Stephen Montgolfier, and the flight covered five miles. The construction of crude dirigibles followed within a few months. The first instance of partial sustentation without the use of gas occurred at this same period. On December 26, 1784, Sebastian Lenormand descended from the tower of the Montpelier Observatory by means of two small parachutes, the idea of the parachute being due to da Vinci. Successful efforts with gliders were made in the last years of the 19th century. Modern aviation dates from 1903, when the Wright brothers first constructed a machine, equipped with engines, which could actually rise from a level field, without the assistance of air currents, and make flights controlled by the pilot. For the history of aëronautics see Albert F. Zahm, "Aërial Navigation," D. Appleton, & Company, New York and London, 1911.

gas, such as hydrogen or helium, affords a means of sustentation. But aërial navigation means far more than the ability to stay aloft, and a craft which can travel only as the wind blows it, can serve few purposes other than that of furnishing amusement. The question of equipping a balloon with engines and a means of propulsion, of traveling in a desired direction with a velocity within our control, of maintaining a desired altitude, of rising and landing, must be answered before we can say the balloon has furnished a means of aërial navigation. It is only since the development of the gasoline motor that this has been possible on any extensive scale.

The second method of attacking the problem seeks to solve simultaneously the problems of sustentation and propulsion. The possibility of propulsion must now come first, for sustentation will be obtained from the motion. It is then evident that this method, even more than that of the balloon, had to await the perfection of a source of energy such as the gasoline engine. It is only with aircraft of this second sort that we are concerned. Such a machine is called an airplane, or aëroplane. We shall adopt the first term. Both names are suggested by the fundamental rôle played by surfaces approximately plane with which the machine is provided. The air reaction on these surfaces, produced by the motion of the machine, furnishes the sustentation. The complete machine will consist of other members, and we can divide it into five distinct parts: the sustaining surfaces, the stabilizing and controlling surfaces, the motor-propeller group, the body, or fuselage, with its place for pilot, passengers and freight, and the landing gear. To these main parts must, of course, be added the various elements of construction by which the different parts are united and the requisite strength given to the complete machine.

We shall not give a discussion of the complete construction of an airplane, but limit ourselves to those features which are necessary for a comprehension of the dynamical problems which we shall study.

2. The principles that govern the construction of an airplane, the phenomena that operate during its flight and determine its behavior, are derived from an understanding of the laws concerning the effect of the wind upon flat and curved surfaces. We can try to determine these laws in two ways, mathematically or experimentally.

In the mathematical method we begin with the principles and equations of hydromechanics. We then see if we can calculate the pressure that a current of air, moving with a certain velocity, will exert upon, for instance, a rectangular plane surface. We must be able to do this for different inclinations of the plane to the air stream. The problem is one of great complexity. In order to construct differential equations that will exhibit the phenomena, and in order to integrate these, we must make assumptions that lead our results to differ from carefully measured observations. For instance, we may assume that the air is a perfect fluid, that is, that it is neither viscous nor compressible. The last assumption seems to be justified for the range of velocities occurring in aëronautical work, but the assumption as to the viscosity vitiates our results when we apply them to actual problems. Even with the assumption of an ideal fluid it is difficult to handle the equations involved. While we can in certain instances obtain information as to how the air streams around obstructing objects, and how it behaves in their vicinity, the results are not such as to make this a simple or satisfactory method of attacking the problem.*

The laws concerning air reaction are determined experimentally in a wind tunnel.† A current of air of several feet thickness is obtained by means of a large fan. Various and accurately known velocities can be given to the air stream. The surfaces upon which we wish to study the pressure are, of course, of limited dimensions, but are similar to those which

* For an elementary mathematical treatment see Cowley and Evans, "Aeronautics in Theory and Experiment," Longmans, 1918, Chapter III.

† Among the various aërodynamical laboratories, can be mentioned those of M. Eiffel at Paris, the National Physical Laboratory of England, the Massachusetts Institute of Technology, and Leland Stanford University. A description of the equipment of such a laboratory can be found in Smithsonian Miscellaneous Collection, Vol. 62.