A TREATISE ON TRILINEAR CO-ORDINATES, INTENDED CHIEFLY FOR THE USE OF JUNIOR STUDENTS

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A Treatise on Trilinear Co-Ordinates, Intended Chiefly for the Use of Junior Students by C. J. C. Price

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C. J. C. PRICE

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A TREATISE

ON

TRILINEAR CO-ORDINATES,

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JUNIOR STUDENTS.

BY

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PREFACE.

THE acknowledged want of a text-book on the subject of Trilinear Co-ordinates, adapted to the use of students for honours in the Mathematical Schools, has led to the publication of the present volume.

It has been found necessary, with a view to rendering the elementary portion of the work as complete as possible, to exclude the consideration of equations of an order higher than the second. For the same reason, problems relating to the focal properties of conic sections have not been discussed; but the Author does not regret that the limits of the work have compelled him to pass over investigations of a class in which little or nothing is gained by the employment of the trilinear method.

The introduction of matter belonging more properly to the department of Pure Geometry, as also any reference to other systems of co-ordinates, has been as far as possible avoided; except, perhaps, in the fifth Chapter, where the importance of the subjects treated and the want of a succinct yet tolerably complete account of them seemed to warrant the digression.

Besides an acquaintance with the principles of the Differential Calculus, such as a student who is about to enter upon this branch of Modern Geometry is sure to possess, the reader is supposed to have some knowledge of the Theory of Determinants.

The Author feels it right to state that the papers on

Trilinear Co-ordinates communicated by Mr. Allen Whitworth, of St. John's College, Cambridge, in the first numbers of the "Messenger of Mathematics" did not come under his notice until the earlier portion of this treatise was written. The results now published were arrived at independently, the perusal of the papers referred to having led only to the insertion of Art. 152.

In writing Chap. V. the Author has derived much assistance from Mr. Townsend's "Modern Geometry" and from a work by M. Housel entitled *Introduction à la Géométrie Supérieure*. To books which are so well known as Dr. Salmon's "Conic Sections" and Mr. Ferrers' "Trilinear Co-ordinates" it is difficult to say to what extent he is indebted.

In conclusion, the Author would take the present opportunity of expressing his sincere thanks to Mr. J. D. Davenport, Fellow of Brasenose College, for his kindness in revising a great portion of the manuscript for the press, as well as for many valuable suggestions, and to other friends for any assistance they may have rendered.

EXETER COLLEGE, OXFORD,
December 4, 1865.

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