ANALYTIC MECHANICS

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Analytic Mechanics by John Anthony Miller & Scott Barrett Lilly

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ANALYTIC MECHANICS

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PREFACE

We have attempted to write a rigorous, teachable introduction to the study of mechanics. We believe that certain fundamental principles of mechanics, used in common by students in the various branches of engineering, in theoretical physics, and in celestial mechanics are essential to the satisfactory progress of a student in any of those fields. To this end, we have chosen as our subject matter only such fundamental theorems. We have not attempted to develop the special methods that are used in some branches of these sciences but not in others, because having fixed the principles on which these methods are based, they are better studied in treatises on those subjects. We have, however, indicated the fields in which certain processes are employed and have told the student where more detailed information can be found.

One purpose of the work is to develop facility in the application of mathematical formulæ to the investigation of physical phenomena. We have observed about the same degree of rigor that is observed in the best modern undergraduate texts in mathematics. We have been careful to state the assumptions that have been made and the conditions which must prevail before a given mathematical process is applicable, and have then proved rigorously the theorems based on the stated conditions.

We have quoted freely the theorems that students who have completed a first course in the calculus are supposed to know. In this sense the book is mathematical.

The illustrative problems that are solved and the exercises which are set for the student are chosen largely from engi-

neering fields. They are real problems chosen from real structures or real machines. In this sense the book is a mechanics of engineering.

We have chosen the older methods of treatment, such as the resolution of forces, etc., because we believe that in this way the student keeps in a little closer touch, at least in the beginning, with the physics of the problem.

While we have tried always to be rigorous, our ambition has been to produce a book that is distinctively teachable. We have made many "remarks" to students, in which we have given directions and cautions in the applications of the fundamental theorems to practical problems. These "remarks" are not mere directions for the solutions of problems, but fundamental discussions of the strength and limitations of basic methods which, once mastered, will be of service throughout the student's professional career.

The book may be adapted to a number of possible courses, emphasizing more or less the engineering or the mathematical phase of the subject, as the reader desires. He may omit, without breaking the logical sequence, any article in fine print, if he omits all succeeding articles in fine print. He may, to reduce the amount of mathematics necessary, omit articles 95–106, from Center of Gravity, and articles 229–242, from Moment of Inertia. If he desires to eliminate parts applicable especially to the analysis of structures, he may omit articles 87–88 (three-force-pieces), and may abbreviate further by omitting articles 85–88. He may omit either all of Chapter iv, or articles 122–124, or all of Chapter viii, or all of Chapter xiii, or any number of these, so far as requisition made on them by succeeding articles is concerned.

We have used $\frac{W}{g}$ instead of "mass."

JOHN A. MILLER.
SCOTT B. LILLY.

SWARTHMORE COLLEGE,
June, 1915.

CONTENTS

INTRODUCTORY

ARTICLE		PAGE
1.	Mechanics Defined	1
2.	Fundamental Elements	2
3.	Mass	2
4.	Length	2
5.	Time	2
6.	Motion	3
7.	Velocity	3
8.	Velocity, either Uniform or Variable	3
9.	Mathematical Expression for Velocity	3
10.	Mathematical Expression for Acceleration	4
11.	Newton's Laws of Motion	4
12.	Discussion of Newton's Laws of Motion	5

PART I. STATICS

CHAPTER I

THE COMPOSITION AND RESOLUTION OF FORCES ACTING ON A PARTICLE

13.	Force	6
14.	Measurement of Force	6
15.	Representation of Force	7
16.	The Resultant	8
17.	Parallelogram of Forces	8
18.	Proofs of Parallelogram of Forces	8
19.	Definitions	9
20.	Analytic Determination of Resultant of Two Forces	9
21.	Resultant of Any Number of Forces	10
22.	Polygon of Forces	10

ARTICLE PAGE
23. Remark on Composition and Resolution of Forces . . 11
24. Rectangular Components of a Force 11
25. Coplanar, Collinear, and Concurrent Forces Defined . . 11
26–27. The Resultant of Any Number of Coplanar Forces Acting on a Particle. The Analytic Expression . . 12
28. *Theorem.* — For Any System of Forces Acting on a Particle, There Is One and Only One Resultant 13
29. The Determination of the Resultant of Forces Acting on a Particle, but not Lying in the Same Plane . . . 14
30. Exercises 16

CHAPTER II

STATICS OF A PARTICLE

31. Statics Defined 18
32. Equilibrium 18
33. Résumé 18
34. Analytic Conditions of Equilibrium of Coplanar Forces Acting on a Particle 18
35. Analytic Conditions of Equilibrium of Non-Coplanar Forces 19
36. Exercises 20
37. Conditions of Equilibrium Considered Graphically . . 21
38–40. Triangle of Forces 21
41. Exercises 23
42. Types of Forces 23
43. Mechanics of a Particle May Be Extended to the Mechanics of a Finite Number of Points Rigidly Connected . . 27
44. Remark on Fictitious Bodies 27
45. Solution of Problems. Suggestions 28
46. Exercises 31

CHAPTER III

FORCES ACTING ON A RIGID BODY

47. Rigid Body Defined 35
48. Additional Assumptions 35
49. Non-Parallel Forces 35
50–51. Resultant of Parallel Forces 36

ARTICLE PAGE
52. Center of Parallel Forces Defined 37
53–54. To Find the Center of Any Number of Parallel Forces . 38
55. Properties of the Center of Parallel Forces 40
56. Exercises 41
57. The Moment of a Force 42
58. Sign of a Moment 42
59. The Composition of Moments 42
60. Couples Defined 44
61. *Theorem.* — The Moment of the Forces of a Couple with Respect to Any Point in the Plane of the Couple is Constant and Equal to the Product of One of the Forces and the Arm of the Couple 44
62. *Theorem.* — The Effect of a Couple on a Rigid Body Is Unchanged if it Be Replaced by Another Couple in the Same or a Parallel Plane, Having the Same Moment . 44
63. Composition of Couples 46
64. The Axis of a Couple 47
65. We May Combine a Couple and a Force Acting in the Same Plane or Parallel Planes into a Single Force . . . 48
66. *Theorem.* — Any Number of Forces Acting on a Rigid Body Is Equivalent to a Force and a Couple 49
67. A System Equivalent to Any Number of Forces Acting on a Rigid Body 51

CHAPTER IV

VECTORS

68. Vectors and Scalars Defined 54
69. Representation of Vectors 54
70. Equivalent Vectors 55
71. Addition of Vectors 55
72. Subtraction of Vectors 56
73. Product of a Vector and a Scalar 57
74. Composition and Resolution of Vectors 57
75. Rectangular Components of a Vector 57
76. Components of a Vector along Three Lines Which Are Mutually Perpendicular 58
77. The Resultant of Any Number of Vectors 58
78. Applications of Theorems on Vectors 59

CHAPTER V

The Statics of a Rigid Body

ARTICLE		PAGE
79.	Conditions for Equilibrium of a System of Coplanar Forces Acting on a Rigid Body	60
80.	An Important Corollary	61
81.	Remark on Solution of Problems	62
82.	Uniform Bar Defined	62
83.	Illustrative Problem	63
84.	Exercises	64
85.	Method of Sections Two-force Pieces	65
86.	Exercises	67
87.	Three-force Pieces	68
88.	Exercises	76

CHAPTER VI

The Center of Gravity

89.	Center of Gravity Defined	82
90.	Definitions	82
91.	Some Special Cases	82
92.	Principles of Symmetry	83
93.	Center of Gravity of Compound Bodies	84
94.	Exercises	86
95.	General Formulæ for Coördinates of Center of Gravity	88
96.	The Weight per Cubic Unit, ρ	89
97.	The Quantity, dV	89
98.	The Quantity, dV (Continued)	91
99.	To Find the Center of Gravity of a Circular Arc	94
100.	Exercises	95
101.	To Find the Center of Gravity of a Quadrant of an Ellipse	95
102.	To Find the Center of Gravity of a Circular Area of Uniform Thickness, the Density of Which Varies as the Distance from a Point on the Circumference	96
103.	To Find the Center of Gravity of the Area of Uniform Density Included between a Parabola and a Straight Line Which Passes through the Vertex	96