MATHEMATICAL TRACTS FOR PHYSICISTS. INTRODUCTION TO THE CALCULUS OF VARIATIONS

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MATHEMATICAL TRACTS FOR PHYSICISTS

INTRODUCTION
TO THE
CALCULUS OF VARIATIONS

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CHAPTER I

INTRODUCTION

1. The Calculus of Variations owed its origin to the attempt to solve a very interesting and rather narrow class of problems in Maxima and Minima, in which it is required to find the form of a function such that the definite integral of an expression involving that function and its derivative shall be a maximum or a minimum.

Let us consider three simple examples: The Shortest Line, The Curve of Quickest Descent, and The Minimum Surface of Revolution.

(*a*) *The Shortest Line.* Let it be required to find the equation of the shortest plane curve joining two given points.

We shall use rectangular coördinates in the plane in question taking one of the points as the origin. Call the coördinates of the second point x_1, y_1.

If $y = f(x)$ is a curve through $(0, 0)$ and (x_1, y_1) and I is the length of the arc between the points, obviously

$$I = \int_0^{x_1} \sqrt{dx^2 + dy^2}$$

or

$$I = \int_0^{x_1} \sqrt{1 + y'^2}\,dx, \tag{1}$$

and we wish to determine the form of the function f so that this integral shall be a minimum.

(*b*) *The Curve of Quickest Descent.* Let it be required to find the form of a smooth curve lying in a vertical plane and joining two