

GRAPHICAL METHODS

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Graphical Methods by Carl Runge

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CARL RUNGE

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GRAPHICAL METHODS

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TABLE OF CONTENTS.

§ 1. Introduction	v
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CHAPTER I. Graphical Calculation.

§ 2. Graphical arithmetic	1
§ 3. Integral functions	6
§ 4. Linear functions of any number of variables	18
§ 5. The graphical handling of complex numbers	25

CHAPTER II. The Graphical Representation of Functions of One or More Independent Variables

§ 6. Functions of one independent variable	40
§ 7. The principle of the slide rule	43
§ 8. Rectangular coördinates with intervals of varying size	52
§ 9. Functions of two independent variables	58
§ 10. Depiction of one plane on another plane	65
§ 11. Other methods of representing relations between three variables	84
§ 12. Relations between four variables	94

CHAPTER III. The Graphical Methods of the Differential and Integral Calculus.

§ 13. Graphical integration	101
§ 14. Graphical differentiation	117
§ 15. Differential equations of the first order	120
§ 16. Differential equations of the second and higher orders	136



INTRODUCTION.

§ 1. A great many if not all of the problems in mathematics may be so formulated that they consist in finding from given data the values of certain unknown quantities subject to certain conditions. We may distinguish different stages in the solution of a problem. The first stage we might say is the proof that the quantities sought for really exist, that it is possible to satisfy the given conditions or, as the case may be, the proof that it is impossible. In the latter case we have done with the problem. Take for instance the celebrated question of the squaring of the circle. We may in a more generalized form state it thus: Find the integral numbers, which are the coefficients of an algebraic equation, of which π is one of the roots. Thirty years ago Lindemann showed that integral numbers subject to these conditions do not exist and thus a problem as old almost as human history came to an end. Or to give another instance take Fermat's problem, for the solution of which the late Mr. Wolfskehl, of Darmstadt, has left \$25,000 in his will. Find the integral numbers x, y, z that satisfy the equation

$$x^n + y^n = z^n,$$

where n is an integral number greater than two. Fermat maintained that it is impossible to satisfy these conditions and he is probably right. But as yet it has not been shown. So the solution of the problem may or may not end in its first stage.

In many other cases the first stage of the solution may be so easy, that we immediately pass on to the second stage of finding methods to calculate the unknown quantities sought for. Or even if the first stage of the solution is not so easy, it may be expedient to pass on to the second stage. For if we succeed in finding methods of calculation that determine the unknown quan-

v