ELEMENTARY PROJECTIVE GEOMETRY

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Elementary projective geometry by A. G. Pickford

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A. G. PICKFORD

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by

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PREFACE

THE development of the methods of Projective Geometry forms an important part of Modern Geometry, and the valuable results obtained justify the increasing attention which is being paid to this subject. I propose, therefore, to arrange in orderly sequence the elementary propositions of plane projective geometry, assuming a knowledge of the first six books of Euclid, or their equivalent, and I trust that this book will be of use to the Upper Forms of Schools, and to Junior Students at the Universities.

The projective unit is the cross-ratio of four collinear points or of four concurrent lines in a plane: from this I proceed to the study of projective rows and pencils, and the involutions of six points or lines, which play an important part in the solution of problems. I then deduce the properties of the curve of the second degree, defined as the locus of the intersections of corresponding rays of two projective pencils, first proving an important harmonic property of the tangent.

The chief properties of polars follow; and of inscribed and circumscribed polygons, with the construction of conics to satisfy five given conditions, and solutions of other problems connected with the conic.

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I conclude with the elements of polar reciprocation, and of plane homology, with brief notes on projection in space and the sections of a circular cone.

In some cases I have developed a point at greater length, as in the extension of Maclaurin's Theorem, and in the treatment of the harmonic conics of four-points and four-sides. The student should draw many figures in addition to those given in the book; the examples given at the ends of the several chapters include many of the questions recently set in this subject, and also propositions suggested by the text, and others chosen from various writers of the last century.

In an elementary treatment of the subject I have avoided dependence on the use of points at infinity and imaginary points and lines; these will find place in a more advanced treatise, and also the properties of curves of degree higher than the second, and of surfaces and curves in space of three dimensions.

In conclusion I must thank the Syndics of the University Press for undertaking the publication of this book, and the Readers of the Press for their carefulness in the revision of the proofs.

A. G. P.

28 *August*, 1909.

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