

# **PLANE AND SPHERICAL TRIGONOMETRY**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649535187

Plane and Spherical Trigonometry by Leonard M. Passano

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.  
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

[www.triestepublishing.com](http://www.triestepublishing.com)

**LEONARD M. PASSANO**

**PLANE AND  
SPHERICAL  
TRIGONOMETRY**



PLANE AND SPHERICAL  
TRIGONOMETRY

BY

LEONARD M. PASSANO

ASSOCIATE PROFESSOR OF MATHEMATICS  
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

New York

THE MACMILLAN COMPANY

1921

*All rights reserved*

✓ Educ T 169.21.675

PRINTED IN THE UNITED STATES OF AMERICA

THIS BOOK IS PROPERTY  
TRANSFERRED FROM THE  
LIBRARY OF THE  
GRADUATE SCHOOL OF EDUCATION  
Oct 26 1927

COPYRIGHT, 1918,  
BY THE MACMILLAN COMPANY.

Set up and electrotyped. Published April, 1918.

FERRIS  
PRINTING COMPANY  
NEW YORK CITY

## PREFACE

Of late years, in the writing of textbooks of trigonometry, a tendency to amplification has shown itself, doubtless with the idea that amplification means simplification. Unfortunately the amplification has spent itself upon details rather than upon principles, which latter have too often been inadequately treated. The result has been textbooks which overlook the comparative maturity of the boys and girls who study trigonometry and which cling almost with affection to the practices of the most elementary mathematics.

The present text aims to present the trigonometry in such a way as to make it interesting to students approaching some maturity, and so as to connect the subject, not only with the mathematics which the student has already had, but also with the mathematics which, in many cases at least, is to follow. A subject may be so burdened with detailed explanations as to become monotonous and lifeless, or, on the other hand, presented in so concise and difficult a manner as to be repellent. The present work endeavors to avoid both extremes. Full explanations are given of important principles, but many simple details are left to the work of the student.

The following points in the text may be noted :

1. Positive and negative angles of any magnitude and the trigonometric functions of such angles, defined by means of a system of rectangular coördinates, are taken up in the beginning of the book ; acute angles, with their functions, being mentioned as a special case.
2. Thus the basic trigonometric identities are got at once for all angles.

3. The functions of  $0^\circ$ ,  $90^\circ$ , etc., are carefully explained by the theory of limits.

4. The solution of right triangles and related problems are taken up early without the use of logarithms.

5. Logarithms are then very carefully explained and fully discussed, not so much as to their use in computation, but rather so as to clarify their meaning.

6. Right triangles are then solved by the use of logarithms, and the essentially approximate nature of all numerical results is emphasized.

7. The text next returns to trigonometric identities, giving a detailed and accurate proof of the addition formulæ for sines and cosines, with less detailed but sufficient explanation of other fundamental identities. The number of identities to be memorized is reduced to a minimum.

8. The circular measure of an angle and the inverse functions are then taken up, emphasis being laid upon the fact that the latter are angles.

9. There follows the solution of triangles in general. As each case is mentioned the theorems or formulæ needed for its solution are derived.

10. The last subject treated in the plane trigonometry is the solution of trigonometric equations, and the fact is emphasized that the operations are simply the solution of algebraic equations applied to a new class of quantities.

11. The lists of examples and problems are numerous and carefully chosen, many of them being taken from work in analytic geometry and calculus, though, of course, no knowledge of either of these subjects is assumed. Some of the problems are entirely new, being invented for this text, and all problems are chosen with a purpose to indicate the practical interest and value of trigonometry.

12. In the spherical trigonometry, as in the plane, the three chief aims are brevity, clarity, and simplicity; a chapter on the Earth treated as a sphere being given to enliven an otherwise somewhat formal and lifeless subject.

13. The author has *not* tried to revolutionize the teaching



of trigonometry, believing that much that has been done in the past is good though none the less open to improvement. Such improvement has been the aim of this work.

The author wishes to acknowledge the kindness of his colleagues Professor H. W. Tyler, Professor F. L. Hitchcock, and Professor J. Lipka in reading and criticizing the manuscript of his book, and to express his thanks to Professor E. R. Hedrick, editor of the tables appended, for permission to make use of them.

L. M. PASSANO.



## CONTENTS

ART.	PAGE
INTRODUCTION . . . . .	xiii
<b>PLANE TRIGONOMETRY</b>	
<b>CHAPTER I. THE TRIGONOMETRIC FUNCTIONS OF ANY ANGLE AND IDENTICAL RELATIONS AMONG THEM .</b>	
	1-13
1. Rectangular coordinates . . . . .	1
2. Angles of any magnitude . . . . .	1
3. Abscissa, ordinate, and distance . . . . .	3
4-5. The trigonometric functions defined . . . . .	4
6. Signs of the functions . . . . .	6
7. Functions of acute angles . . . . .	7
8. Reciprocal functions . . . . .	7
9. Tangent, sine, and cosine . . . . .	8
10. Sine and cosine . . . . .	9
11. Tangent and secant . . . . .	9
12-13. Fundamental relations, collected . . . . .	10
14. Values of the functions when one is given . . . . .	10
<b>CHAPTER II. IDENTICAL RELATIONS AMONG THE FUNCTIONS OF RELATED ANGLES. THE VALUES OF THE FUNCTIONS OF CERTAIN ANGLES . . . . .</b>	
	14-27
15. Functions of negative angles . . . . .	14
16. Functions of $90^\circ - \alpha$ . . . . .	15
17. Functions of $90^\circ + \alpha$ . . . . .	16
18. Functions of $180^\circ + \alpha$ . . . . .	17
19. Generalization . . . . .	18
20. Functions of certain angles . . . . .	20
21. Functions of $30^\circ$ and $60^\circ$ . . . . .	20
22. Functions of $45^\circ$ . . . . .	21
23. Functions of $120^\circ$ , $135^\circ$ , etc. . . . .	22
24. Functions of $0^\circ$ . . . . .	22
25-26. Functions of $90^\circ$ , etc. . . . .	25
27. Limiting values of functions . . . . .	26