

# **NEW PLANE GEOMETRY**

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New Plane Geometry by Wooster Woodruff Beman & David Eugene Smith

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**WOOSTER WOODRUFF BEMAN & DAVID EUGENE SMITH**

# **NEW PLANE GEOMETRY**



NEW  
PLANE GEOMETRY

BY

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## PREFACE.

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THE demand of a large number of schools for a book confined to plane geometry leads the authors to issue this edition of the first part of their "New Plane and Solid Geometry." In offering it to the profession an explanation of its distinctive features may be of service.

It is sometimes asserted that we should break away from the formal proofs of Euclid and Legendre and lead the student to independent discovery, and so we find text-books that give no proofs, others that give hints of the demonstrations, and still others that draw out the demonstration by a series of questions which, being capable of answer in only one way, merely conceal the Euclidean proof. But, after all, the experience of the world has been that the best results are secured by setting forth a minimum of formal proofs as models, and a maximum of unsolved or unproved propositions as exercises. This plan has been followed by the authors, and the success of the first edition has abundantly justified their action.

There is a growing belief among teachers that such of the notions of modern geometry as materially simplify the ancient should find place in our elementary text-books. With this belief the authors are entirely in sympathy. Accordingly they have not hesitated to introduce the ideas of one-to-one correspondence, of anti-parallels, of negative magnitudes, of general figures, of similarity of point systems, and such other concepts as are of real value in the early

study of the science. All this has been done in a conservative way, and such material as the first edition (1895) showed to be at all questionable has been omitted from the present revision.

Within comparatively recent years the question of methods of attack has interested several leading writers. Whatever has been found to be usable in elementary work the authors have inserted where it will prove of most value. To allow the student to grope in the dark in his efforts to discover a proof, is such a pedagogical mistake that this innovation in American text-books has been generally welcomed. Upon this point the authors have freely drawn from the works of Petersen of Denmark, and of Rouché and de Comberousse of France, and from the excellent treatise recently published by Hadamard (Paris, 1898).

With this introduction of modern concepts has necessarily come the use of certain terms and symbols which may not generally be recognized by teachers. These have, however, been chosen only after most conservative thought. None is new in the mathematical world, and all are recognized by the leading writers of the present time. They certainly deserve place in our elementary treatises on the ground of exactness, of simplicity, and of their general usage in mathematical literature.

The historical notes of the first edition have been retained, it being the general consensus of opinion that they add materially to the interest in the work. For teachers who desire a brief but scholarly treatment of the subject the authors refer to their translation of Fink's "History of Elementary Mathematics" (Chicago, The Open Court Publishing Co., 1899). For the limitations of elementary geometry, the impossibility of trisecting an angle, squaring a

circle, etc., teachers should read the authors' translation of Klein's valuable work, "Famous Problems of Elementary Geometry" (Boston, Ginn & Company).

It is impossible to make complete acknowledgment of the helps that have been used. The leading European text-books have been constantly at hand. Special reference, however, is due to such standard works as those of Henrici and Treutlein, "Lehrbuch der Elementar-Geometrie," the French writers already mentioned, and the noteworthy contributions of the recent Italian school represented by Faifofer, by Socci and Tolomei, and by Lazzeri and Bassani.

Teachers are urged to consider the following suggestions in using the book :

1. Make haste slowly at the beginning of each book.
2. Never attempt to give all of the exercises to any class. From a third to a half, selected by the teacher, should suffice.
3. Require frequent written work, thus training the eye, the hand, and the logical faculty together. The authors' Geometry Tablet (Ginn & Company) is recommended for this work.

W. W. DEMAN, ANN ARBOR, MICH.

D. E. SMITH, BROCKPORT, N. Y.

AUGUST 15, 1899.



10

11

12

13

14

15

16

17

18

19

20

21

22

# CONTENTS.

## INTRODUCTION.

	PAGE
1. ELEMENTARY DEFINITIONS . . . . .	1
2. THE DEMONSTRATIONS OF GEOMETRY . . . . .	9
3. PRELIMINARY PROPOSITIONS . . . . .	13

## BOOK I. — RECTILINEAR FIGURES.

1. TRIANGLES . . . . .	21
2. PARALLELS AND PARALLELOGRAMS . . . . .	43
3. PROBLEMS . . . . .	67
4. LOCI OF POINTS . . . . .	80

## BOOK II. — EQUALITY OF POLYGONS.

1. THEOREMS . . . . .	90
2. PROBLEMS . . . . .	109
3. PRACTICAL MENSURATION . . . . .	112

## BOOK III. — CIRCLES.

DEFINITIONS . . . . .	114
1. CENTRAL ANGLES . . . . .	116
2. CHORDS AND TANGENTS . . . . .	119
3. ANGLES FORMED BY CHORDS, SECANTS, AND TANGENTS . . . . .	128
4. INSCRIBED AND CIRCUMSCRIBED TRIANGLES AND QUADRILATERALS . . . . .	136
5. TWO CIRCLES . . . . .	143
6. PROBLEMS . . . . .	146

## APPENDIX TO BOOK III.

	PAGE
METHODS . . . . .	153

## BOOK IV. — RATIO AND PROPORTION.

1. FUNDAMENTAL PROPERTIES . . . . .	159
2. THE THEORY OF LIMITS . . . . .	167
3. A PENCIL OF LINES CUT BY PARALLELS . . . . .	170
4. A PENCIL CUT BY ANTIPARALLELS OR BY A CIRCUMFERENCE . . . . .	177
5. SIMILAR FIGURES . . . . .	182
6. PROBLEMS . . . . .	194

BOOK V. — MENSURATION OF PLANE FIGURES.  
REGULAR POLYGONS AND THE CIRCLE.

1. THE MENSURATION OF PLANE FIGURES . . . . .	199
2. THE PARTITION OF THE PERIGON . . . . .	205
3. REGULAR POLYGONS . . . . .	209
4. THE MENSURATION OF THE CIRCLE . . . . .	216

## APPENDIX TO PLANE GEOMETRY.

1. SUPPLEMENTARY THEOREMS IN MENSURATION . . . . .	226
2. MAXIMA AND MINIMA . . . . .	229
3. CONCURRENCE AND COLLINEARITY . . . . .	238

## TABLES.

NUMERICAL TABLES . . . . .	245
BIOGRAPHICAL TABLE . . . . .	246
TABLE OF ETYMOLOGIES . . . . .	249
INDEX . . . . .	253