

**INSTRUMENT VARIABLE
ESTIMATION OF
MISSPECIFIED MODELS. WP
#1508-83, DECEMBER 1983**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649739172

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Abstract

This paper studies the estimation of models in which the set of instruments is not, in fact, orthogonal to the residuals. I first show that, in overidentified models of this type, one can generally obtain arbitrary estimates by varying the weights given to different instruments. I then weaken the assumptions of instrumental variable estimation by allowing for nondegenerate price distributions over the product of instruments and residuals. If the variance covariance matrix of this distribution is diagonal, the estimates which minimize the impact of misspecification are shown to lie inside the polyhedron of estimates from the exactly identified submodels.

Introduction

Consider the single equation model:

$$Y = X\beta + \epsilon \quad (1)$$

where Y is a $T \times 1$ vector, X a $T \times k$ matrix, β a $k \times 1$ vector of parameters of interest and ϵ a $T \times 1$ vector of disturbances. Often economic reasoning predicts that ϵ_t is uncorrelated with a series of variables Z_{it} (which may include X 's). It is then natural to estimate the vector β by the method of instrumental variables proposed by Reiersol (1945), discussed in detail in Sargan (1958) and generalized by Hansen (1982). This method considers the sample inner products of the instruments and residuals $Z_i'(Y-X\beta)$ where Z_i is the vector of observations on instrument i . It then sets k linear combinations of these products equal to zero so that

$$WZ'(Y-X\beta) = 0 \quad (2)$$

where Z is a $T \times m$ matrix of instruments, $m \geq k$ and W is a $k \times m$ weighting matrix of rank k .

The hypotheses that the expected value of $Z_{it}\epsilon_t$ is exactly zero is probably false for most economic models. This explains in part why, in empirical papers this hypothesis is often rejected by Hausman (1978) tests and other specifications tests. In particular, such rejections are reported by: Hansen and Singleton (1983) Mankiw, Rotemberg and Summers (1982), Pindyck and Rotemberg (1983).¹ After all, the models are only an approximation to reality. The lack of concern expressed over these rejections must mean that the authors imagine on a priori grounds that the inconsistency of the

resulting estimates must be small. This belief may be based on Fischer's (1961) "proximity theorem," which states that for a fixed W as the mean of $\epsilon_t' Z_{it}$ goes to zero the inconsistency of β disappears in a continuous fashion. This paper argues that this optimism may be unfounded. I show that when overidentified models (i.e. models where $m \geq k$) are misspecified even slightly, the estimated β 's may be extremely far from the true β 's. This result does not contradict Fischer's result directly. This is so because I keep the mean $\epsilon_t' Z_{it}$ fixed and I consider changes in the weighting matrix W . If the means of $\epsilon_t' Z_{it}$ differ sufficiently across instruments, one can obtain essentially arbitrary $\hat{\beta}$'s by varying W .

Methods have been proposed for selecting weighting matrices that minimize the asymptotic covariance matrix of the β 's under the assumption that the model is correctly specified. In particular if the ϵ_t 's are i.i.d. then the "optimal" W is $X'Z(Z'Z)^{-1}$ and the resulting estimator is obtained by two stage least squares. Here I propose a different estimation procedure. This procedure is designed to minimize the impact of misspecification. I assume that $Z'_i \epsilon / T$ converges to V_i as T goes to infinity. However, instead of assuming V_i is zero, I treat V_i as an unknown random variable from the point of view of the econometrician. I assume that V_i has mean zero and variance σ_i^2 (so that, on average the estimates are consistent). Also the expected value of $V_i V_j$ is zero so that the asymptotic biases from the different instruments are uncorrelated. Under these circumstances I discuss the instrumental variables estimator which minimizes the asymptotic covariance matrix of $\hat{\beta}$. I show that this optimal $\hat{\beta}$ is strictly inside the polyhedron whose vertices are obtained from estimating the $\begin{matrix} m \\ n \end{matrix}$ exactly

