ELEMENTS OF GEOMETRY

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Elements of geometry by Andrew W. Phillips & Irving Fisher

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ANDREW W. PHILLIPS & IRVING FISHER

ELEMENTS OF GEOMETRY



PHILLIPS-LOOMIS MATHEMATICAL SERIES

ELEMENTS OF GEOMETRY

BY

ANDREW W. PHILLIPS, Ph.D.

ANI

IRVING FISHER, PH.D.

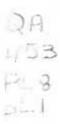
PROFESSORS IN YALE UNIVERSITY

PART ONE-PLANE GEOMETRY



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PREFACE

THE present volume consists of the first five books of the authors' "Elements of Geometry," or that portion which relates to Plane Geometry.

While the book speaks for itself, we would call attention to some of its most important features.

The Introduction presents in the shortest possible compass the general outlines of the science to be studied, and leads at once to the actual study itself.

The definitions are distributed through the book as they are needed, instead of being grouped in long lists many pages in advance of the propositions to which they apply. An alphabetical index is added for easy reference.

The constructions are also distributed, so that the student is taught how to make a figure at the same time that he is required to use it in demonstration.

Extensive use has been made of natural and symmetrical methods of demonstration. Such methods are used for deducing the formula for the sum of the angles of a triangle, for the sum of the exterior and interior angles of a polygon, for parallel lines, for the theorems on regular polygons, and for similar figures.

The theory of limits is treated with rigor, and not passed over as self-evident.

Attention is also called to the theorems of proportion and the use of corollaries as exercises to supply the need of "inventional geometry." We would here express our grateful acknowledgments to all who have aided in the preparation of this book; to Miss Elizabeth H. Richards, whose successful experience in fitting students for college in Plane Geometry has rendered her criticisms and suggestions most valuable; and to our colleagues, Messrs. W. M. Strong and Joseph Bowden, Jr. Mr. Strong has selected, for the most part, the exercises at the end of the book, and Mr. Bowden has examined critically the references and proof-sheets of the book.

> Andrew W. Phillips, Irving Fisher.

YALE UNIVERSITY.

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SPECIAL TERMS

An axiom is a truth assumed as self-evident.

A theorem is a truth which becomes evident by a train of reasoning called a demonstration,

A theorem consists of two parts, the Appothesis, that which is given, and the conclusion, that which is to be proved.

A problem is a question proposed which requires a solution.

A proposition is a general term for either a theorem or problem.

One theorem is the converse of another when the conclusion of the first is made the hypothesis of the second, and the hypothesis of the first is made the conclusion of the second.

The converse of a truth is not always true. Thus, "If a man is in New York City he is in New York State," is true; but the converse, "If a man is in New York State he is in New York City," is not necessarily true.

When one theorem is easily deduced from another the first is sometimes called a corollary of the second.

A theorem used merely to prepare the way for another theorem is sometimes called a lemma.

SYMBOLS AND ABBREVIATIONS

+ plus. Cons.—Construction.

- minus. Cor.—Corollary.

> is greater than. Def.—Definition.

< is less than. Fig.—Figure.

× multiplied by. Hyp.—Hypothesis.

= equals. Iden.—Identical.

□ is equivalent to. Q. E. D.—Quod erat demonstrandum.

Alt.-int.—Alternate interior. Q. E. F.—Quod erat faciendum.

Ax.—Axiom. Sup.-adj.—Supplementary adjacent.