ELEMENTS OF GEOMETRY

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ELEMENTS OF GEOMETRY

BY

ANDREW W. PHILLIPS, PH.D.

AND

IRVING FISHER, PH.D.

PROFESSORS IN YALE UNIVERSITY

PART ONE—PLANE GEOMETRY

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PREFACE

THE present volume consists of the first five books of the authors' "Elements of Geometry," or that portion which relates to Plane Geometry.

While the book speaks for itself, we would call attention to some of its most important features.

The *Introduction* presents in the shortest possible compass the general outlines of the science to be studied, and leads at once to the actual study itself.

The *definitions* are distributed through the book as they are needed, instead of being grouped in long lists many pages in advance of the propositions to which they apply. An alphabetical index is added for easy reference.

The *constructions* are also distributed, so that the student is taught how to make a figure at the same time that he is required to use it in demonstration.

Extensive use has been made of *natural* and *symmetrical* methods of demonstration. Such methods are used for deducing the formula for the sum of the angles of a triangle, for the sum of the exterior and interior angles of a polygon, for parallel lines, for the theorems on regular polygons, and for similar figures.

The *theory of limits* is treated with rigor, and not passed over as self-evident.

Attention is also called to the theorems of *proportion* and the use of *corollaries* as *exercises* to supply the need of "inventional geometry."

We would here express our grateful acknowledgments to all who have aided in the preparation of this book; to Miss Elizabeth H. Richards, whose successful experience in fitting students for college in Plane Geometry has rendered her criticisms and suggestions most valuable; and to our colleagues, Messrs. W. M. Strong and Joseph Bowden, Jr. Mr. Strong has selected, for the most part, the exercises at the end of the book, and Mr. Bowden has examined critically the references and proof-sheets of the book.

ANDREW W. PHILLIPS,
IRVING FISHER.

YALE UNIVERSITY.

TABLE OF CONTENTS

PAGE

INTRODUCTION

FUNDAMENTAL CONCEPTIONS 3
GEOMETRIC AXIOMS 4
GENERAL AXIOMS 5

BOOK I

FIGURES FORMED BY STRAIGHT LINES 6
PARALLEL LINES AND SYMMETRICAL FIGURES 15
TRIANGLES 32
PARALLELOGRAMS 60
PROBLEMS 66

BOOK II

THE CIRCLE 73
MEASUREMENT 87
LIMITS 88
PROBLEMS OF DEMONSTRATION 106
PROBLEMS OF CONSTRUCTION 108

BOOK III

PROPORTION AND SIMILAR FIGURES 110
TRANSFORMATION OF PROPORTIONS 111
PROBLEMS OF DEMONSTRATION 154
PROBLEMS OF CONSTRUCTION 158
PROBLEMS FOR COMPUTATION 160

BOOK IV

AREAS OF POLYGONS 170
PROBLEMS OF DEMONSTRATION 193

PAGE
PROBLEMS OF CONSTRUCTION 196
PROBLEMS FOR COMPUTATION. 197

BOOK V

REGULAR POLYGONS AND CIRCLES. SYMMETRY WITH RESPECT TO A POINT 202
PROBLEMS OF DEMONSTRATION 229
PROBLEMS OF CONSTRUCTION 230
PROBLEMS FOR COMPUTATION. 231

EXERCISES

BOOK I 235
BOOK II 236
BOOK III 233
BOOK IV 238
BOOK V 239

INDEXES

DEFINITIONS 253
CONSTRUCTIONS 256
RULES OF MENSURATION 257

SPECIAL TERMS

An **axiom** is a truth assumed as self-evident.

A **theorem** is a truth which becomes evident by a train of reasoning called a **demonstration.**

A theorem consists of two parts, the *hypothesis*, that which is given, and the *conclusion*, that which is to be proved.

A **problem** is a question proposed which requires a solution.

A **proposition** is a general term for either a theorem or problem.

One theorem is the **converse** of another when the conclusion of the first is made the hypothesis of the second, and the hypothesis of the first is made the conclusion of the second.

The converse of a truth is not always true. Thus, "If a man is in New York City he is in New York State," is true; but the converse, "If a man is in New York State he is in New York City," is not necessarily true.

When one theorem is easily deduced from another the first is sometimes called a **corollary** of the second.

A theorem used merely to prepare the way for another theorem is sometimes called a **lemma.**

SYMBOLS AND ABBREVIATIONS

+ plus.	Cons.—Construction.
− minus.	Cor.—Corollary.
> is greater than.	Def.—Definition.
< is less than.	Fig.—Figure.
× multiplied by.	Hyp.—Hypothesis.
= equals.	Iden.—Identical.
≂ is equivalent to.	Q. E. D.—Quod erat demonstrandum.
Alt.-int.—Alternate interior.	Q. E. F.—Quod erat faciendum.
Ax.—Axiom.	Sup.-adj.—Supplementary adjacent.