

**ELEMENTS OF DESCRIPTIVE
GEOMETRY: WITH APPLICATIONS
TO SPHERICAL AND ISOMETRIC
PROJECTIONS, SHADES AND
SHADOWS, AND PERSPECTIVE**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649571161

Elements of Descriptive Geometry: With Applications to Spherical and Isometric Projections,
Shades and Shadows, and Perspective by Albert E. Church & George M. Bartlett

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

www.triestepublishing.com

ALBERT E. CHURCH & GEORGE M. BARTLETT

**ELEMENTS OF DESCRIPTIVE
GEOMETRY: WITH APPLICATIONS
TO SPHERICAL AND ISOMETRIC
PROJECTIONS, SHADES AND
SHADOWS, AND PERSPECTIVE**

ELEMENTS
OF
DESCRIPTIVE GEOMETRY

QA
501
C561
1911

WITH APPLICATIONS TO
SPHERICAL AND ISOMETRIC PROJECTIONS, SHADES AND
SHADOWS, AND PERSPECTIVE

BY

ALBERT E. CHURCH, LL.D.

LATE PROFESSOR OF MATHEMATICS IN THE UNITED STATES
MILITARY ACADEMY

AND

GEORGE M. BARTLETT, M.A.

INSTRUCTOR IN DESCRIPTIVE GEOMETRY AND MECHANISM
IN THE UNIVERSITY OF MICHIGAN

NEW YORK .. CINCINNATI .. CHICAGO
AMERICAN BOOK COMPANY

Engin
GA
501
CS61
1911

COPYRIGHT, 1864, BY
BARNES & BUHR.

COPYRIGHT, 1864, BY
A. S. BARNES & CO.

COPYRIGHT, 1892 AND 1902, BY
MARGARET A. BLUNT.

COPYRIGHT, 1911, BY
GEORGE M. BARTLETT.

ENTRÉD AT STATIONERS' HALL, LONDON.

G. E. DEBCK, GRAM,
W. P. 6

PREFACE

CHURCH'S "Elements of Descriptive Geometry" was originally published in 1864. The preface to the first edition states: "Without any effort to enlarge or originate, the author has striven to give, with a natural arrangement and in clear and concise language, the elementary principles and propositions of this branch of science, of so much interest to the mathematical student, and so necessary to both the civil and military engineer."

Professor Church succeeded so well in his efforts to produce a practical and well-adapted treatise that it has continued in use as a text-book for more than forty years in the United States Military Academy and in many other academies, technical schools, and colleges. This long-continued use of the book speaks well for its high intrinsic excellence.

During the last few years, however, there have taken place many changes in the methods of teaching the subject, and in the problems required. To meet these new demands the present volume is issued. In its preparation much of Professor Church's text has been used, and his concise and lucid style has been preserved.

Among the salient features of the present work are the following:

The *figures* and *text* are included in the same volume.

General cases are preferred to special ones.

A sufficient number of problems are solved in the *third angle* to familiarize the student with its use.

A treatment of the *profile plane* of projection is introduced.

Many *exercises for practice* have been introduced.

Several *new problems* have been added.

The *old figures have been redrawn*, and many of them have been improved.

Several of the more difficult elementary problems have been illustrated by *pictorial views*.

In the treatment of *curved surfaces*, all problems relating to single-curved surfaces are taken up first, then those relating to warped surfaces, and finally those relating to surfaces of revolution. Experience proves this order to be a logical one, as we here proceed "from the simple to the more complex." Also the student is more quickly prepared for drawing-room work on intersections and developments; and in case it is desired to abbreviate the course by omitting warped surfaces, the remaining problems will be found to be consecutively arranged.

The writer here wishes to acknowledge his indebtedness to the many teachers who have aided him with valuable advice and suggestions in relation to this work. In particular his thanks are due to his esteemed colleagues, Professor H. J. Goulding and Mr. D. E. Foster of the University of Michigan, for their careful reading and correction of the manuscript.

G. M. B.

MAY 14, 1900.

CONTENTS

PART I

ORTHOGRAPHIC PROJECTIONS

	PAGE
Preliminary Definitions	7
Representation of Points	9
Representation of Planes	11
Representation of Straight Lines	11
Propositions relating to the Point, Line, and Plane	12
Rotation of the Horizontal Plane	17
Notation used in the Description of Drawings	19
Exercises for Practice	21
The Profile Plane of Projection	25
Elementary Problems relating to the Point, Line, and Plane	28
Classification of Lines	63
Projection of Curves	64
Tangents and Normals to Lines	65
Construction of Certain Plane Curves	69
The Helix. Generation and Properties	73
Generation and Classification of Surfaces	77
Cylindrical Surfaces. Generation and Properties	78
Conical Surfaces. Generation and Properties	81
Planes Tangent to Surfaces in General	84
Planes Tangent to Cylinders and Cones	86
Points in which Surfaces are pierced by Lines	94
Intersection of Cylinders and Cones. Developments	96
Convolutcs, and Problems relating to Them	120
Warped Surfaces with a Plane Director	125
The Hyperbolic Paraboloid	128
Planes Tangent to Warped Surfaces with a Plane Director	132
The Helicoid	138
Warped Surfaces with Three Linear Directrices	142
Surfaces of Revolution	146
The Hyperboloid of Revolution of One Nappe	147
Double-Curved Surfaces of Revolution	156
Planes Tangent to Surfaces of Revolution	158
Intersection of Surfaces of Revolution with Other Surfaces	163
Problems relating to Trihedral Angles. Graphical Solution of Spherical Triangles	169

PART II

SPHERICAL PROJECTIONS

	PAGE
Preliminary Definitions	179
Orthographic Projections of the Sphere	182
Stereographic Projections of the Sphere	189
Globular Projections	201
Gnomonic Projection	203
Cylindrical Projection	203
Conic Projection	203
Construction of Maps	205
Lorgna's Map	206
Mercator's Chart	207
Flamstead's Method	208
The Polyconic Method	210

PART III

SHADES AND SHADOWS

Preliminary Definitions	212
Shadows of Points and Lines	215
Construction of an Ellipse on its Conjugate Diameters	219
Practical Problems	220
Brilliant Points	231

PART IV

LINEAR PERSPECTIVE

Preliminary Definitions and Principles	238
Perspectives of Points and Straight Lines. Vanishing Points of Straight Lines	239
Perspectives of Curves	243
Vanishing Points of Rays of Light and of Projections of Rays	247
Perspectives of the Shadows of Points and Straight Lines on Planes	248
Practical Problems	250

PART V

ISOMETRIC DRAWING

Preliminary Definitions and Principles	278
Isometric Representation of Points and Lines	280
Practical Problems	281

PART I

ORTHOGRAPHIC PROJECTIONS

PRELIMINARY DEFINITIONS

1. Geometry enables us to determine unknown magnitudes, relationships, and forms from those which are known. There are in general two methods of solution for any given problem; namely, the *analytical* and the *graphical*. In the former we arrive at our results by calculation; in the latter we make drawings which represent graphically the true relationships between the points, lines, and surfaces under consideration, and arrive at our results without calculation.

2. Graphics. If the problem relates to points and lines lying in only one plane, the graphical solution may be reached by a simple application of the principles of Geometrical Drawing, or *Plane Graphics*.

If the problem relates to magnitudes not in the same plane, the graphical solution would require an application of the principles of Descriptive Geometry, or the *Graphics of Space*.

3. Descriptive Geometry is that branch of Mathematics which has for its object the explanation of the methods of representing by drawings:

First. All geometrical magnitudes.

Second. The solution of problems relating to these magnitudes in space.

These drawings are so made as to present to the eye, situated at a particular point, the same appearance as the magnitude or object itself, were it placed in the proper position.