CAMBRIDGE PROBLEMS PROPOSED BY THE MODERATORS TO THE CANDIDATES FOR MATHEMATICAL HONORS AT THE GENERAL EXAMINATION, 1843

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Cambridge problems proposed by the moderators to the candidates for mathematical honors at the general examination, 1843 by Various

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Trieste

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TO THE

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AT THE

GENERAL EXAMINATION, 1843.

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CAMBRIDGE PROBLEMS.

WEDNESDAY, *January* 4, 1843.

QUESTIONS IN PURE MATHEMATICS.

N.B.—The Differential Calculus is not to be used.

1. THE opposite sides and angles of parallelograms are equal to one another, and the diameter bisects them, that is, divides them into two equal parts.

2. In a circle the angle in a semi-circle is a right angle; but the angle in a segment greater than a semi-circle is less than a right angle; and the angle in a segment less than a semi-circle is greater than a right angle.

3. What sum must be invested in the 3 per cent. stock at $94\frac{1}{4}$, to yield an annual income of £500?

4. Shew how to find the least common multiple of two quantities: and extend the method to find that of several quantities.

Ex. Find the least common multiple of $x^3 - x$, $x^3 - 1$, $x^3 + 1$.

5. Find the sum of any number of terms of a series in arithmetic progression.

The sum of an arithmetic progression is 72, the first term is 24, and the common difference -4, find the number of terms.

6. Solve the equations:

(1) $\frac{2x-6}{5} - \frac{x-4}{9} - \frac{3x}{13} = 0.$ (2) $x^4 - 14x^2 + 1 = 0.$

(3) $$\begin{aligned} a\,x + b\,y + c\,z &= d, \\ a_1 x + b_1 y + c_1 z &= d_1, \\ a_2 x + b_2 y + c_2 z &= d_2. \end{aligned}$$

Find when equations (3) are equivalent to only two independent equations.

7. The coefficients of terms equidistant from the ends, in the expansion of $(1 + x)^n$ (n a positive integer) are equal.

Write down the general term of $\frac{1}{\sqrt{1+x}}$.

8. Shew that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$.

Prove that $\tan(45^0 + A) - \tan(45^0 - A) = 2 \tan 2A$. What is the use of the latter formula?

9. Explain the ambiguous case of plane triangles, giving a geometrical illustration of it. When $C = 30^0$, $b = 16$, $c = 8$, is the triangle ambiguous or not?

10. Find the angle between two straight lines of which the equations are given.

Find the equations to lines passing through a given point, and inclined at an angle of 45^0 to the line $y = x + a$.

11. Find the equation to a straight line touching a parabola at a given point.

Find the points of contact of tangents, the perpendiculars on which from the foot of the directrix, shall be equal to one-fourth of the latus rectum.

12. Define conjugate diameters of an ellipse. Shew that there can be but one pair of rectangular conjugate diameters.

13. Shew how to reduce the reciprocal equation

$$x^{2n} - px^{2n-1} + \ldots = 0$$

which has no roots equal to unity.

If the terms be alternately positive and negative, and p not greater than $2n$, the roots cannot all be real.

14. In isosceles spherical triangles, the angles at the base are equal.

15. Prove the formula for finding the cosine of an angle of a spherical triangle in terms of the sides.

16. Shew how to construct a table of natural sines.

Given cosine $34^\circ.29' = 4338835$, cosine $34^\circ.30' = 4335938$.

Find the cosine of $34^\circ.29'.48''$.

WEDNESDAY, *January* 4.

QUESTIONS IN NATURAL PHILOSOPHY.

N.B.—The Differential Calculus is not to be used.

1. ASSUMING the parallelogram of forces, determine the resultant of any number of forces in the same plane acting on a point.

At any point of the circumference of a circle two equal forces act in directions passing through two fixed points on the circumference, shew that the resultant of these forces always passes through a fixed point.

2. Find the ratio of the power to the weight when there is equilibrium on the wheel and axle.

What mechanical powers are reducible to the principle of the lever?

3. Define the centre of gravity of a body. Find the centre of gravity of a quadrilateral figure two sides of which are parallel.

4. How is force measured in Dynamics? What is denoted by the letter (g) in dynamical formulæ? If the unit of time be one minute, what is the measure of the accelerating force of gravity? How are the Statical and Dynamical measures of force connected?

5. State the principles on which is founded the theory of the direct impact of elastic bodies. Two equal and imperfectly elastic bodies impinge directly: having given the original velocity of the one, find that of the other, in order that after the blow it may be reduced to rest.

6. Determine the directions in which a particle must be projected with a given velocity, in order that it may hit a mark at a given distance on the same horizontal line with the point of projection.

7. A body floats in water, find the conditions of equilibrium.

A cylinder with its axis vertical floats in two fluids of different densities, find the ratio of the two parts into which the cylinder is divided by the common surface of the two fluids.

8. Describe the common low pressure steam-engine. Is it necessary that the steam should be admitted into the cylinder during the whole stroke?

9. By what experiments is it proved that the pressure of air is proportional to its density, the temperature being constant? If the temperature vary, what is the law of variation of the pressure?

10. Shew that the image of an object in a plane mirror is equal to the object, and equally inclined to the mirror.

11. Find the limit of the refracting angle of a prism of given refractive power so that the passage of a ray of light through it may be possible.

12. Describe the construction of the simple astronomical telescope, shewing on what depend its magnifying power, and the brightness of the image.