NOTES ON RANKINE'S APPLIED MECHANICS, PART I; AND RANKINE'S CIVIL ENGINEERING, PARTS II & III

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Notes on Rankine's Applied Mechanics, Part I; And Rankine's Civil Engineering, Parts II & III
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NOTES ON RANKINE'S APPLIED MECHANICS, PART I; AND RANKINE'S CIVIL ENGINEERING, PARTS II & III

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NOTES

ON

Rankine's Applied Mechanics _ Part I.

AND

Rankine's Civil Engineering _ Parts II & III.

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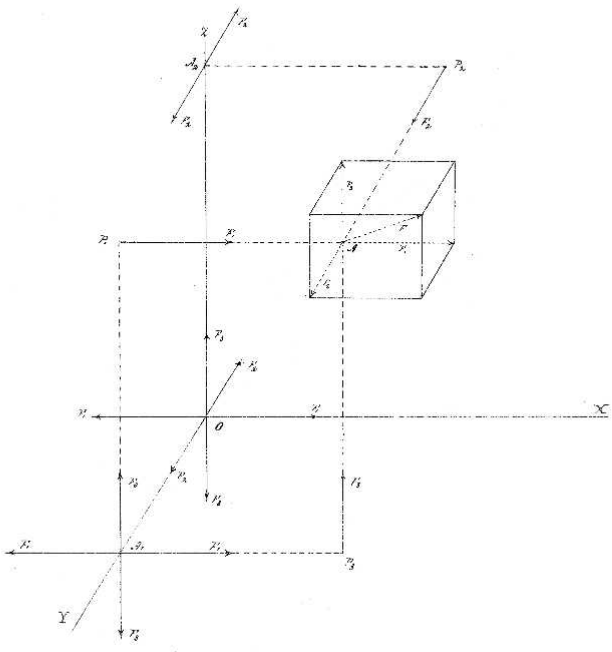
Rankine's Applied Mechanics
Art. 60.

To determine the resultant of any number of forces having any directions and magnitudes & any points of application whatever in space.



This is most readily done by reffering all the forces to three coordinate axes at right angles to each other with origin at the most convenient point. Let OX, OY & OZ be these axes, & let the forces in the directions of X, Y & Z from O be considered positive & those in the opposite directions negative. Also let the rotations which appear "righthanded" when looking from X, Y and Z towards O be considered positive (that is motion from X to Y, from Y to Z & from Z to X) & the contrary ones negative.

Let F be any one of the forces & A its point of application in space. Resolve this force into three compts. parallel to the three axes. Then if α, β and γ be the angles made by F with OX, OY & OZ respectively, we shall have for the compts.

$$F\cos\alpha = F_1$$
$$F\cos\beta = F_2$$
$$F\cos\gamma = F_3$$

Now, let us transfer each of these compt. forces to the origin & find its equivalent. The force, F_1, may be considered as applied at any point in its line of direction, &, consequently, may be considered as applied at the point, P_1, where its line of direction pierces the plane, ZY. Drop a perp. from P_1 upon the axis, OY. It meets it at A_1. At this point (A_1) apply two equal & opposite forces, each parallel & equal to F_1. They do not alter the condition of the body, but the left one of these two forces considered in connection with the force F_1 at P_1 constitutes a couple whose force $= F_1$ & whose lever-arm is P_1A_1 = the coordinate, z, of A. There remains the other force $= F_1$ applied at A_1. Hence, F_1 at A is equivalent to F_1 at A_1 & a couple $= +F_1z$.

Again the force F_1 applied at A_1 may be transferred to O; for, if we apply at O two equal & opposite forces, each parallel & equal to F_1, we shall have F_1 applied at A_1 equivalent to F_1 applied at O plus a couple whose force $= F_1$ & whose lever-arm $= A_1O = y$ (one of the coordinates of A) & which is negative since it tends to produce rotation from Y towards X

This couple ∴ $= -F_1 y$. Hence to sum up. The compᵗ F_1 of F parallel to OX & applied at A is equivalent to an equal & parallel force applied at the origin plus the two couples $+F_1 z$ $-F_1 y$.

Now let us take the compᵗ of F in the direction of OY. This $= F_2$. Prolong its line of direction until it meets the plane ZX at P_2. From this point drop a perp. upon OZ, & at A_2 apply two opposite forces each equal & parallel to F_2. The force F_2 at P_2 is = then to the force F_2 applied at A_2 & a couple $+F_2 x$. Again the force F_2 at A_2 is = an equal & parallel force at O plus a couple whose value $= -F_2 z$. Hence finally the compᵗ F_2 at A is equivalent to a force F_2 at O plus the two couples $+F_2 x$ and $-F_2 z$. In the same way by considering the third compᵗ F_3 of F as applied at P_3 & then transferring it to A_1 & thence to O we find it may be replaced by a force F_3 applied at O & two couples $+F_3 y$ and $-F_3 x$. Hence the force F applied at A is equivalent to the three forces

$$\left.\begin{matrix} F_1 \\ F_2 \\ F_3 \end{matrix}\right\} \text{applied at } O \text{ along the respective axes}$$

and six couples

$$\left.\begin{matrix} F_1 z & -F_1 y \\ F_2 x & -F_2 z \\ F_3 y & -F_3 x \end{matrix}\right\} \text{each set of two corresponding to one of the comp}^{ts} \text{ forces.}$$

But, instead of arranging the couples in sets with regard to the compᵗ forces (F_1, F_2, F_3) it is more convenient to group them with reference to the axes around which they tend to produce rotation. Grouped in this way they may be written

$F_3 y$	$-F_2 z$	tending to produce rotation around OX
$F_1 z$	$-F_3 x$	" " " " " OY
$F_2 x$	$-F_1 y$	" " " " " OZ

A similar analysis may be made for all the forces.

and by summing results we have for the resultants of any number of forces in space

$$\left.\begin{aligned} \Sigma F\cos\alpha &= \Sigma F_1 = R_1 = \text{resultant of components along } OX \\ \Sigma F\cos\beta &= \Sigma F_2 = R_2 = \quad " \quad " \quad " \quad " \quad OY \\ \Sigma F\cos\gamma &= \Sigma F_3 = R_3 = \quad " \quad " \quad " \quad " \quad OZ \end{aligned}\right\} (1)$$

and

$$\left.\begin{aligned} \Sigma(F\cos\gamma . y - F\cos\beta . z) &= \Sigma(F_3 y - F_2 z) = M_1 = \text{sum of couples about } OX \\ \Sigma(F\cos\alpha . z - F\cos\gamma . x) &= \Sigma(F_1 z - F_3 x) = M_2 = \quad " \quad " \quad " \quad " \quad OY \\ \Sigma(F\cos\beta . x - F\cos\alpha . y) &= \Sigma(F_2 x - F_1 y) = M_3 = \quad " \quad " \quad " \quad " \quad OZ \end{aligned}\right\} (2)$$

The above resultant forces, R_1, R_2, R_3, acting at O may be combined into one force (The diag. of the parallelopipedon constructed on them), & the value of this final resultant will be

$$R = \sqrt{R_1^2 + R_2^2 + R_3^2} \text{ - - - - - - - - (4)}$$

and if α_r, β_r, γ_r, be the angles it makes with the axes

$$\cos\alpha_r = \frac{R_1}{R}, \quad \cos\beta_r = \frac{R_2}{R}, \quad \cos\gamma_r = \frac{R_3}{R}$$

These eq.s make known the magnitude & direction of the resultant force, its point of application being O.

Similarly, the three couples M_1, M_2, M_3 may be combined into one whose value is

$$M = \sqrt{M_1^2 + M_2^2 + M_3^2} \text{ - - - - - - - - (5)}$$

and if λ, μ, ν are the angles made by the axis of the resultant couple, M, with the axes of the component couples (which are the coordinate axes themselves) we shall have

$$\cos\lambda = \frac{M_1}{M}, \quad \cos\mu = \frac{M_2}{M}, \quad \cos\nu = \frac{M_3}{M}$$

Thus the value of the resultant couple and the direction of its axis are determined.

We thus see that any number of forces acting in space may be replaced by a single force $= R$ acting at the origin, whose magnitude & direction are determined, plus a couple M also of determinate magnitude whose plane of action is known, since it is perp. to its axis.

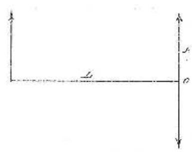
The conditions of equilibrium of such a system of forces are

$$R = 0\,, \quad M = 0$$

When the system is not balanced its resultant must fall under one of the five following cases –

1° When $M = 0$, the resultant is the single force R, acting at O.

2° When the axis of M is at right angles to the direction of R, then M acts in a plane identical with or parallel to that in which R acts. If the plane of M is parallel to that of R, we may consider the couple as transferred to the plane of R, and then we have a couple & a force in one plane to deal with. If the forces of this couple are not each $= R$, change it into an



equivalent couple with forces $= R$. & move the couple so that one of its forces shall be applied at O & opposite to R. Then it is plain from the figure that the couple & force in one plane are equivalent to a single force $= R$ applied at a distance from $O = L = \frac{M}{R}$, to the left if the couple is righthanded & to the right if the couple is left-handed.

The condition rendering the axis of M perp. to R is

$$\cos\alpha_r \cos\lambda + \cos\beta_r \cos\mu + \cos\gamma_r \cos\nu = 0$$

3° When $R = 0$, the only resultant is the couple M & we have rotation without translation.

4° When the axis of M is parallel to R, then the couple M acts in a plane perp. to R & there can be no farther reduction. The body under such a system of forces will rotate in a plane perp. to the line of direction in which it moves forward (like a rifle ball). The condition rendering the axis of M parallel to R is

$\lambda = \alpha_r$ or $= -\alpha_r$; $\mu = \pm\beta_r$; $\nu = \pm\gamma_r$.

5° When the axis of M is oblique to R making with it an angle given by the eq.

$$\cos\theta = \cos\lambda\cos\alpha_r + \cos\mu\cos\beta_r + \cos\nu\cos\gamma_r .$$

The couple M may be resolved into two components; viz- $M\sin\theta$ around an axis perp. to R & in a plane containing the direction of R & of the axis of M, & $M\cos\theta$ around an axis parallel to R. The force R & the couple, $M\sin\theta$, are equivalent, as in Case (2), to a single force equal & parallel to R whose line of action is in a plane perp. to that containing R & the axis of M, & whose perp. distance from O is

$$L_1 = \frac{M\sin\theta}{R}$$

The couple, $M\cos\theta$, which acts in a plane perp. to R is incapable of further combination. Hence every system of forces not in equilibrium is equivalent to a single force ((1) & (2)) or to a couple (3) or to a force & a couple ((4) & (5)).

Cor. I Suppose the points of application of the forces to be all in one plane - that of XY, for example. Then eq.s (1) & (4) remain unchanged - that is -

$$R_1 = \Sigma.F\cos\alpha = \Sigma.F_1$$
$$R_2 = \Sigma.F\cos\beta = \Sigma F_2 \qquad R = \sqrt{R_1^2+R_2^2+R_3^2}$$
$$R_3 = \Sigma.F\cos\gamma = \Sigma F_3$$

But in eq.s (2) the coordinates, z, $= 0$ or

$$M_1 = \Sigma(F\cos\gamma . y) = \Sigma.F_3 y$$
$$M_2 = -\Sigma(F\cos\gamma . x) = -\Sigma F_3 x$$
$$M_3 = \Sigma(F\cos\beta . x - F\cos\alpha . y) = \Sigma(F_2 x - F_1 y)$$

and

$$M = \sqrt{M_1^2+M_2^2+M_3^2}$$

Cor. II Suppose the forces all lie in one plane.