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THE PRECESSIONAL CONSTANT

WASHINGTON
BUREAU OF EQUIPMENT, NAVY DEPARTMENT
1897

To the first of these is frequently applied the term "Constant of Precession." But as it is a variable quantity, the application of the term "constant" to it might be misleading. Indeed, no one of the four quantities just described can be regarded as a fundamental constant in astronomy, not only because they all vary, but because at least two of them are a result of both motions—that of the ecliptic and that of the equator.

In the case of the lunisolar precession, defined as the movement of the equinox upon a fixed ecliptic, there is some ambiguity. As the term is used by BESSEL and others, the fixed ecliptic is that of 1750, no matter for what epoch the precession may be defined. The annual lunisolar precession then diminishes with the time. But if we consider the precession upon the fixed ecliptic of the date, then the annual value is a quantity which increases with the time. When we have removed this ambiguity by adopting one or the other of these definitions, we may regard the lunisolar precession as dependent solely upon the motion of the equator, and as defining that motion. The same motion is defined by the quantity n, which expresses its annual value when measured on a great circle of the sphere. On the other hand, the two general precessions are resultants of both motions—that of the ecliptic and that of the equator.

To avoid the confusion which may thus arise in treating the subject, I have proposed the term "Precessional Constant" for a certain function P of the masses of the Sun, Earth, and Moon, and of the elements of the orbits of the Earth and Moon, on which the motion of the equator depends. This quantity changes only through the diminution of the Earth's eccentricity, and the change is so slight that it can become appreciable only after several centuries. From the value of P and the motion of the ecliptic, the four motions of the precession are derived by the formulæ,

General precession; $p = P \cos \varepsilon - \varkappa \sin L \cos \varepsilon$
Lunisolar precession; $p_1 = P \cos \varepsilon$
Precession in R. A.; $m = P \cos^2 \varepsilon - \varkappa \sin L \operatorname{cosec} \varepsilon$
Precession in declination; $n = P \cos \varepsilon \sin \varepsilon$

Here L is the longitude of the instantaneous axis of rotation of the ecliptic, $\varkappa$ is its annual motion, and ε is its obliquity.

SECTION I.

Previous determinations.

Of the authoritative determinations of the precessional motion made during the nineteenth century the first is that of BESSEL. It rests upon a comparison of the star catalogue of the *Fundamenta Astronomiæ* with the catalogue of PIAZZI for 1800, the latter being first corrected by more recent Königsberg observations. The values of the motions as they are quoted in ENGELMANN's edition of BESSEL's Abhandlungen from the Memoirs of the Berlin Academy of Sciences for 1815* are found below. But

*BESSEL's Abhandlungen, von R. Engelmann, Vol. I, pp. 262–285.

in the Tabulæ Regiomontanæ, published in 1830, somewhat different values are given. The two sets of values are as follows:

	Abhandlungen.	Tabulæ Regiomontanæ.
	$''$ $''$	$''$ $''$
Gen. prec.;	$50.18828 + .02442966T$	$50.22350 + .02443T$
Lunisolar;	$50.32832 - .02435890$	$50.36354 - .02436$
	$m = 46.01135 + .03086450$	$46.04367 + .03086$
	$n = 20.04554 - .00970204$	$20.05957 - .00970$

T is here counted from 1800.0. in terms of the century as the unit.

The interval between the two catalogues on which this determination rested was only forty-five years. In 1840, the shortness of this period and the fact that much material had accumulated for improving the values of the constants in question, led OTTO STRUVE to make a new determination. This determination, with some alterations by PETERS, has been most in use to the present time. It depends upon a comparison of the positions of 400 stars in the *Fundamenta* with determinations made principally at Dorpat about the mean epoch 1825. It therefore rests upon an interval of about seventy years.* The marked feature of this determination was that it included a determination of the solar motion, which was thus eliminated from the result. The corrections to the provisional value of the motion of precession for seventy years, which I suppose to be that of BESSEL, were found to be:

In R. A.; $\Delta m = +1''.6743$; $\Delta p = +1''.16 \pm 0''.67$
In Decl.; $\Delta n = +0''.2624$; $\Delta p = +0''.66 \pm 0''.86$
The combined result is $\Delta p = +0''.97 \pm 0''.53$.
$p = 50''.23492$ for the epoch 1790.

In the Astronomische Nachrichten, No. 485, STRUVE the elder quotes the result as

$$50''.23449 \pm 0''.00771$$

Neither of the STRUVES give any precessional motion but this. The motions with which STRUVE's name is associated are found in the memoir of PETERS, *Numerus Constans Nutationis*. Here, on page 160, PETERS gives 50''.3798 as the lunisolar precession for 1800 found by STRUVE. He afterwards deduces the following values of the various quantities on which the processional motions depend, as those resulting from STRUVE'S work:

General precession, 1800; $50.2411 + .02268T$
Lunisolar precession, 1800; $50.3798 - .02168$
$m = 46.0623 + .02849$
$n = 20.0607 - .00863$

These values are tabulated on page 200 of the paper, and are those in most general use up to the present time. It will, however, be seen that the value of the general

*Mémoires de l'Académie Impériale des Sciences de Saint-Pétersbourg, sixième série. Sciences Mathématiques et Physiques, Tome III.

precession is larger by 0″.43 per century than that found by STRUVE in his printed paper. I am not aware of the origin of this change, but its effects was to increase the error of a determination already somewhat too large.

As I pointed out to the Conference in 1896, some confusion has arisen from the fact that in applying PETERS's formulæ to the trigonometric reduction of star positions the obliquity of the equator to the fixed ecliptic has been carried only to terms in t^2, whereas, to make the formulæ consistent and homogeneous, this quantity should be carried to terms in t^3.

Before the above values had entirely superseded those of BESSEL, suspicion arose that they were too large, and that those of BESSEL were really nearer the truth, although the data on which they rested were much inferior. In his "Annales de l'Observatoire," Vol. II, LE VERRIER adopts the following values:

$$\begin{aligned} &\text{General precession (1850);} \quad 50''.23572 + ''.022578T \\ &\text{Lunisolar precession;} \quad 50.37140 - .021762 \\ &m = 46.06010 + .028373 \\ &n = 20.05240 - .008663 \end{aligned}$$

These have been used in the reductions of stars at the Paris Observatory.

In his "*Lehrbuch zur Bahnbestimmung der Kometen und Planeten,*" Vol. I second edition, OPPOLZER develops the most exhaustive theory of precession which has yet appeared, but without any discussion of the observations on which his adopted values of the constants depend. His results are (pp. 202, 206):

$$\begin{aligned} &\text{General precession (1850);} \quad 50''.23465 + ''.022580T \\ &\text{Lunisolar precession;} \quad 50.36924 - .021776 \\ &m = 46.05931 + .028390 \\ &n = 20.05150 - .008668 \end{aligned}$$

About 1886 LUDWIG STRUVE took advantage of the completion of AUWERS' reduction of BRADLEY's observations, and of the POULKOWA catalogue for 1845, 1855, and 1865, to enter upon the most thorough discussion of the observations which has yet been published. The negative correction found to OTTO STRUVE's result was quite large. The precessional motions m and n, found by L. STRUVE, were

$$\begin{aligned} &\text{General precession (1850);} \quad 50''.2283 + ''.02120T \\ &\text{Lunisolar precession;} \quad 50.3407 - .02132 \\ &m = 46.0554 + .02741 \\ &n = 20.0452 - .00849 \end{aligned}$$

It must be stated, however, that the smallness of these values arises partly from the large positive correction which AUWERS applied to BRADLEY's Right Ascensions.

When the various determinations are reduced to a common equinox, it is found that the discordances are materially diminished.*

All the preceding determinations rest upon BRADLEY stars, which I conceive to afford the best material available at the present time. Determinations from a great number of faint stars have been made by DREYER, NYRÉN, and BOLTE.

DREYER's investigation is found in the Journal *Copernicus*, Volume II, Dublin, 1882. It is prefaced by a very exhaustive history of previous determinations of this constant. The material consists of LALANDE's catalogue, and the catalogue of SCHJELLERUP for 1865. The system of right ascensions to which both catalogues are reduced is N_1. The results for $1800 + t$ are—

$$\text{Lunisolar precession} = 50''.3752 - 0''.0002168t$$
$$\text{General precession} = 50.2365 + 0.0002268t$$
$$m = 46.0581 + 0.0002849t$$
$$n = 20.0589 - 0.0000862t$$

NYRÉN's result was based on a comparison of stars in BESSEL zones with SCHJELLERUP, and is much smaller than the above. The motion is, however, greatly increased when the two catalogues are reduced to the system N_1, so that, ultimately, the discordance is not great. But the brevity of the interval renders the determination weak.

BOLTE's investigation is based on the declinations of LALANDE as compared with SCHJELLERUP. His work was published only as a doctorial thesis, and is therefore not so well known as it should be.

These determinations would be better than any that could be made from BRADLEY stars were it not for the probable systematic errors in the observations and reductions. First among these errors is that in Right Ascension depending on the magnitude of the star. When we come down to faint stars this error may increase to an almost unknown extent. Moreover, as a general remark applicable to all zone observations, it may be said that there is great room for systematic errors, both in the Right Ascension and Declination, which would materially affect the value of the precessional motion derived from them.

List of the principal determinations.

For the sake of easy comparison the values above given, with some others, are reduced to 1850, and the results shown below.

In the reductions of the earlier values to 1850 the motion in my "Elements and Constants" has been used.

The value N_0 is the preliminary one, used in the tables of the planets and in my "Elements and Constants."

*See Vierteljahrsschrift der Astronomischen Gesellschaft, Vol. XIII, 1878, page 107.

The value N_{96} is that reached in the present paper.

	General precession.	m.	n.
	"	"	"
Bessel I,	50.1994	46.0253	20.0413
Bessel II,	50.2346	46.0576	20.0553
Struve,	50.2478		
Peters,	50.2522	46.0763	20.0564
Leverrier,	50.2357	46.0601	20.0524
Oppolzer,	50.2346	46.0593	20.0515
L. Struve,	50.2283	46.0554	20.0452
N_0,	50.2371	46.0636	20.0479
Dreyer,	50.2478	46.0723	20.0546
Nyrén,	50.1995	46.0426	
Bolte,			20.0537
N_{96},	50.2453	46.0711	20.0511

Section II.

Material used in the present discussion.

In starting with a new determination, the first question to be considered is what material we shall use. What we want is the annual motion of the equinox, arising from the combined motions of the equator and the ecliptic, relative to an axis absolutely fixed in space. As observations can not be referred to any line or plane which we know to be absolutely fixed, we are obliged to assume the general mean position of the fixed stars as affording the invariable axes of reference. Here we meet with difficulties arising from the proper motions of the stars, and especially from the solar motion. The elimination of the latter from the mean result is a matter of especial difficulty.

It is also to be recalled that the position of the equinox relative to the mean stellar sphere, assumed as fixed, requires three practically separate determinations. These are—

1. The correction to the Sun's absolute mean right ascension or longitude. This is obtained principally from observations of the Sun's declination.

2. The correction to the general mean right ascension of the clock or standard stars relative to the Sun.

3. The determination of the positions of the clock stars relative to the great mass of stars assumed to define the fixed celestial sphere.

In the present investigation it is needless to consider the first two determinations because, by a resolution of the Conference of 1896, in accordance with which this investigation is made, the position of the equinox among the clock stars is defined by the system N_1, formed by those equatorial stars whose position is given in the Appendix to the *Catalogue of* 1098 *Clock and Zodiacal Stars*, found in Volume I of these *Astronomical Papers*. It is therefore only necessary, so far as the Right Ascensions are concerned, to determine the positions of the great mass of stars relative to the standard stars in question.

The question now arises, what stars we shall choose. Undoubtedly, the great mass of the fainter stars, down to the ninth magnitude, which we find in various zones observed during the present century, would be preferable. But a determination made from these stars would be subject to very large systematic error, arising from the shortness of the interval between the earlier and the later observations. This uncertainty is magnified by the possibility of very large personal error due to the small magnitude of the star. It is now well known that, at the present time, especially when observations are made by the chronograph, faint stars are observed too late relatively to brighter ones. Were this personal error constant at all times and for all observers during the century, no error would thus arise. But there is strong reason for suspecting that this error not only varies from one observer to another, but that it is much larger for the recent than for the older observations. If such is the case, the precessional motion derived from such a comparison would be too great. The probability of this error has led me to depend mainly upon observations of the brighter stars, especially those in BRADLEY's catalogue. As the total number of stars exceeds 3,000, it may be expected that all accidental errors will be nearly eliminated from the mean. Though the systematic error just considered may not be wholly eliminated, it will certainly be much smaller than in the case of a determination founded on much fainter stars observed during a shorter interval.

I have, however, made an exception of the LALANDE catalogue by using the comparisons of this catalogue with those of BOSS and SCHJELLERUP. The former determined his personal error dependent on magnitude. In the case of SCHJELLERUP this error seems to be small.

SECTION III.

Provisional precessions.

From the considerations already set forth, the desirableness will be seen of adopting at the start a provisional value of the precessional constant which will be as near as practicable to the truth. That STRUVE's value needs a large negative correction is now widely admitted, and, it seems to the writer, made quite clear by the data already set forth in the preceding pages. I shall, therefore, take as provisional values the precessional motion already alluded to, which is employed in my "Elements and Constants," as the provisional value to be corrected.

The principal quantities for the epoch 1850 are these:

The precessional constant for a solar century of 36524.2 days

$$P = 5489''.78 - 0''.00364T$$

$$\begin{aligned} &\qquad\qquad\qquad\qquad {}'' \\ \text{General precession;}\quad p &= 5023.71 \\ \text{Lunisolar precession;}\; p_1 &= 5036.02 \\ 100\,m &= 4606.36 \\ 100\,n &= 2004.79 \end{aligned}$$

The fundamental quantity to be determined is the correction to this value of P; but it will be more convenient to take p_1, the lunisolar precession, as the quantity to be