THE ELEMENTS OF THE DIFFERENTIAL CALCULUS: COMPREHENDING THE GENERAL THEORY OF CURVE SURFACES, AND OF CURVES OF DOUBLE CURVATURE

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ELEMENTS

OF THE

DIFFERENTIAL CALCULUS;

COMPREHENDING THE

GENERAL THEORY OF CURVE SURFACES,

AND OF

CURVES OF DOUBLE CURVATURE.

INTENDED FOR THE USE OF

MATHEMATICAL STUDENTS IN SCHOOLS AND UNIVERSITIES.

BY J. R. YOUNG,

AUTHOR OF

"THE ELEMENTS OF ANALYTICAL GEOMETRY."

REVISED AND CORRECTED, BY

MICHAEL O'SHANNESSY, A.M.

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1833.

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ADVERTISEMENT.

This edition of Young's Differential and Integral Calculus is presented to the American public, with a confidence in its favourable reception, proportionate to that which the original acquired in England. The text has not been materially altered, though many errors have been corrected, some of which by Professor Dodd of Princeton College, N. J.

These volumes will be found to contain a full elementary course of the subject of which they treat, and well adapted as a text book for Colleges and Universities.

The second volume, treating exclusively of the Integral Calculus, is now in press, and will be speedily published.

New-York, March, 1833.

PREFACE.

The object of the present volume is to teach the principles of the *Differential Calculus*, and to show the application of these principles to several interesting and important inquiries, more particularly to the general theory of Curves and Surfaces. Throughout these applications I have endeavoured to preserve the strictest rigour in the various processes employed, so that the student who may have hitherto been accustomed only to the pure reasoning of the ancient geometry will not, I think, find in these higher order of researches any principle adopted, or any assumption made, inconsistent with his previous notions of mathematical accuracy. If I have, indeed, succeeded in accomplishing this very desirable object, and have really shown that the applications of the Calculus do not necessarily involve any principle that will not bear the most scrupulous examination, I may, perhaps, be allowed to think that I have, in this small volume, contributed a little towards the perfecting of the most powerful instrument which the modern analysis places in the hand of the mathematician.

It is the adoption of exceptionable principles, and even, in some cases, of contradictory theories, into the elements of this science, that have no doubt been the chief causes why it has hitherto been so little studied in a country where the ancient geometry has been so extensively and so successfully cultivated. The student who proceeds from the works of *Euclid* or of *Apollonius* to study those of our modern analysts, will be naturally enough startled to find that in the *theory* of the Differential Calculus he is to consider that as absolutely *nothing* which, in the *application* of that theory, is to be considered a quantity *infinitely small*. He will naturally enough be startled to find that a conclusion is to be taken as general, when he is at the same time told that the process which led to that conclusion has failing cases; and yet one or both of these inconsistencies pervade more or less every book on the Calculus which I have had an opportunity of examining.

The whole theory of what the French mathematicians vaguely call *consecutive points* and *consecutive elements*, involves the first of these objectionable principles;* for, if the abscissa of any point be represented by x, then the abscissa of the consecutive point, or that separated from the former by an infinitely small interval, is represented by $x + dx$, although dx, at the outset of the subject, is said to be 0. Again, the theory of tangents, the radius of curvature, principles of osculation, &c., are all made to depend upon Taylor's theorem, and therefore can strictly apply only at those points of the curve where this theorem does not fail: the conclusions, however, are to be received in all their generality.†

* It is to be regretted that terms so vague and indefinite should be introduced into the *exact sciences;* and it is more to be regretted that English elementary writers should adopt them merely because they are used by the French, and that too without examining into the import these terms carry in the works from which they are copied. In a recent production of the University of Cambridge, the author, in attempting to follow the French mode of solving a certain problem, has confounded *consecutive points* with *consecutive elements*, two very distinct things: although neither very intelligible, the consequence of this mistake is, that the result is not what was intended; so that, after the process is fairly finished, a new counterbalancing error is introduced, and thus the solution righted!

† I am anxious not to be misunderstood here, and shall therefore state specifically the nature of my objection. In establishing the theory of contact, &c., by aid of Taylor's theorem, it is assumed that a value may be given to the increment h so small as to render the term into which it enters greater than all the following terms of the series taken together. Now how can a function of absolutely indeterminate quantities be shown to be greater or less than a series of other functions of the same indeterminate quantities without, at least, assuming some determinate relation among them? If we say that the assertion applies, whatever particular value we substitute for the indeterminate in the proposed functions or differential coefficients, we merely shift the dilemma, for an indefinite number of these particular values may render the functions all infinite; and we shall be equally at a loss to conceive how one of these infinite quantities can be greater or less than the others. It appears, therefore, that the usual process by which the theory of contact is established, applies rigorously only to those points of curves for which Taylor's development does not fail, and I cannot help thinking that on these grounds the *Analytical Theory of Functions, by Lagrange*, in its application to Ge-

If this statement be true, it is not to be wondered at that students so often abandon the study of this science, less discouraged with its difficulties than disgusted with its inconsistencies. To remove these inconsistencies, which so often harass and impede the student's progress, has been my object in the present volume; and, although my endeavours may not have entirely succeeded, I have still reason to hope that they have not entirely failed. The following brief outline will convey a notion of the extent and pretensions of the book; a more detailed enumeration of the various topics treated of, will be found in the table of contents.

I have taken for the basis of the theory the method of *limits* first employed by *Newton*, although designated by foreign writers as the *method of d'Alembert.* I consider this method to be as unexceptionable as that of *Lagrange*, and on account of its greater simplicity, better adapted to elementary instruction.

The First Chapter is devoted to the exposition of the fundamental principles; and in explaining the notation I have been careful to impress upon the student's mind that the differentials dx, dy, &c. are in themselves absolutely of no value, and that their ratios only are significant: this is the foundation of the whole theory, and it has been adhered to throughout the volume, without any shifting of the hypothesis.

In the Second Chapter it is shown, that if fx represent any function of x, and x be changed into $x + h$, the new state $f(x + h)$ of the function may always be developed according to the ascending integral powers of the increment h; and this leads to the important conclusion that the coefficient of the second term in the development of the function $f(x + h)$ is the differential coefficient derived from the function fx; a fact which *Lagrange* has made the foundation of his

ometry is defective, although I feel anxious to express my opinion of that celebrated performance with all becoming caution and humility. Indeed Lagrange himself has admitted this defect, and observes, (*Théorie des Fonctions, p.* 181,) "Quoique ces exceptions ne portent aucune atteinte à la théorie générale, il est nécessaire, pour ne rien laisser à desirer, de voir comment elle doit être modifier dans les cas particuliers dont il s'agit." (*See note C at the end.*) But he has not modified the expression deduced from this exceptionable theory for the radius of curvature, which indeed is always applicable whether the differential coefficients become infinite or not, although, for reasons already assigned, the process which led to it restricts its application to particular points.

theory of analytical functions. The chapter then goes on to treat of the differentiation of the various kinds of functions, algebraic and transcendental, direct and inverse, and concludes with an article on successive differentiation.

The Third Chapter is devoted to *Maclaurin's theorem*, and its application is shown in the development of a great variety of functions. Occasion is taken, in the course of this chapter, to introduce to the student's attention some valuable analytical formulas and expressions from *Euler*, *Demoivre*, *Cotes*, and other celebrated analysts, together with those curious properties of the circle discovered by *Cotes* and *Demoivre*.

The Fourth Chapter is on *Taylor's theorem*, which makes known the actual development of the function $f(x + h)$ according to the form established in the second chapter. From this theorem are derived commodious expressions for the total differential coefficient when the function is complicated, and whether its form be explicit or implicit; the whole being illustrated by a variety of examples.

The Fifth Chapter contains the complete theory of vanishing fractions.

The Sixth is on the maxima and minima values of functions of a single variable, and will, I think, be found to contain several original remarks and improved processes.

Chapter the Seventh is on the differentiation and development of functions of two independent variables. The usual method of obtaining the development of a function of two variables according to the powers of the increments, is to develop first on the supposition that x only varies and that y is constant, and afterwards to consider y, which is *assumed* to enter into the coefficients, to be changed into $y + h$. But y may be so combined with x in the function $F(x, y)$ that it shall, when considered as a constant, disappear from all the differential coefficients, which circumstances should be pointed out and be shown not to affect the truth of the result: I have, however, avoided the necessity of showing this, by proceeding rather differently. The chapter concludes with *Lagrange's Theorem*, concisely demonstrated and applied to several examples.

The Eighth Chapter completes the theory of maxima and minima, by applying the principles delivered in chapter VI. to functions of two independent variables, and it also contains an important article on