

**ELEMENTS OF DESCRIPTIVE  
GEOMETRY, WITH ITS APPLICATIONS  
TO SPHERICAL PROJECTIONS, SHADES  
AND SHADOWS, PERSPECTIVE AND  
ISOMETRIC PROJECTIONS**

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Elements of descriptive geometry, with its applications to spherical projections, shades and shadows, perspective and isometric projections by Albert E. Church

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**ALBERT E. CHURCH**

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AND SHADOWS, PERSPECTIVE AND  
ISOMETRIC PROJECTIONS**



ELEMENTS  
OF  
DESCRIPTIVE GEOMETRY,

WITH ITS APPLICATIONS TO

SPHERICAL PROJECTIONS, SHADES AND SHADOWS,  
PERSPECTIVE AND ISOMETRIC PROJECTIONS.

BY

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ELEMENTS OF THE DIFFERENTIAL AND INTEGRAL CALCULUS,  
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C. D. G.

## PREFACE.

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THESE pages have been written and are published with the single object of presenting, in proper form to be used as a text-book, the course of Descriptive Geometry, as taught at the U. S. Military Academy.

Without any effort to enlarge or originate, the author has striven to give, with a natural arrangement and in clear and concise language, the elementary principles and propositions of this branch of science, of so much interest to the mathematical student, and so necessary to both the civil and military engineer.

Though indebted for many of the ideas to the early instructions of his predecessor and friend, Professor Davies, whose text-books on this subject were among the first in the English language, the author has been much aided by a frequent reference to the French works of Leroy and Olivier, and to the elaborate American work of Professor Warren.

It is intended to include, in an edition to be issued at an early day, the application of the subject to shades and shadows, and perspective.

U. S. MILITARY ACADEMY, }  
October, 1834 }

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