

**A TREATISE ON STATICS:  
CONTAINING THE THEORY OF  
THE EQUILIBRIUM OF FORCES  
AND NUMEROUS EXAMPLES**

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A Treatise on Statics: Containing the Theory of the Equilibrium of Forces and Numerous Examples by S. Earnshaw

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**S. EARNSHAW**

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*Alexander Zivich*

A TREATISE  
ON  
STATICS,

CONTAINING THE

THEORY OF THE EQUILIBRIUM OF FORCES:

AND

NUMEROUS EXAMPLES

ILLUSTRATIVE OF THE GENERAL PRINCIPLES OF THE SCIENCE.

By *Samuel* S. EARNSHAW, M.A.

OF ST JOHN'S COLLEGE, CAMBRIDGE.

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## PREFACE TO THE THIRD EDITION.

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THIS edition of the Treatise on Statics differs so little from the last as scarcely to require a separate notice. A few Articles have been added, on the pressure which a rigid body is made to exert on a fixed point or axis of support by the action of forces when there is equilibrium. These will be found useful in those Problems of Dynamics wherein it is required to find the pressure which a rigid body in motion, under the influence of any forces, exerts on a fixed point or axis. Indeed, it is chiefly with a view to this application of them that the articles alluded to have been introduced into this edition of the Statics.

The collection of Problems for practice given at the end of the Treatise has been considerably enlarged, chiefly by the addition of Examples of an elementary character. In the selection of them care has been taken to choose such as illustrate Statical Principles under every important variation of aspect, without impeding the student's progress through them by analytical and other difficulties foreign to the proper object of this Treatise.

CAMBRIDGE, Feb. 1, 1845.

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## PREFACE TO THE SECOND EDITION.

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THOUGH the general plan and arrangement of this edition of the Treatise on Statics are the same as in the former, in the details there will be found, it is hoped, some important improvements.

The fundamental proposition of the science,—the Parallelogram of Forces,—I have proved after Duchayla's method, by reason of its simplicity; but I think it necessary here to inform the reader that, as that method is inapplicable when the forces act upon a single particle of matter (as a particle of a fluid medium on the hypothesis of finite intervals), on account of its assuming the transmissibility of the forces to other points than that on which they act, I have, in an Appendix, given the proof which in the first edition was given in the text. The same objection, (and for the same reason) lies against the proof of the parallelogram of forces from the properties of the lever. This method, though allowable in the infancy of the science, can never be exclusively adopted in a treatise which professes to take a more philosophical view of the subject; for, were the transmissibility of force *not* true in fact, the law of the composition of forces acting on a point would still be true; it is evident, therefore, that to make the truth of the former an essential step in the proof of the latter, is erroneous in principle.



In the former edition, forces were considered as acting in any directions in space; a mode of treatment of the subject which necessarily rendered the investigations useless to such readers as had not studied Geometry of Three Dimensions. In the present edition this defect is remedied; and a chapter, in which the forces are supposed to act in a plane, is always made to precede the more general investigations. At the end of Chapter IV. several propositions are proved which have hitherto been used in Elementary Books without proof.

The fifth Chapter contains a new (and it is hoped a satisfactory) and complete proof of the Principle of Virtual Velocities, and its Converse. The proof given by Lagrange in his *Mécanique Analytique*, page 22, et seq., though highly ingenious, I regard as a fallacy; and, if not fallacious, deficient in generality.

In the last Chapter, I have endeavoured to set before the reader such problems as, without involving analytical difficulties, seemed best calculated to make him acquainted with the mode of applying all the most important principles of the science: and not unfrequently I have added remarks upon important steps with the view of pressing them more particularly upon the reader's attention.

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