

THE OUTLINES OF QUATERNIONS

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The Outlines of Quaternions by H. W. L. Hime

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H. W. L. HIME

**THE OUTLINES OF
QUATERNIONS**

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BY
LIEUT.-COLONEL H. W. L. HIME
(LATE) ROYAL ARTILLERY

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