THE OUTLINES OF QUATERNIONS

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The Outlines of Quaternions by H. W. L. Hime

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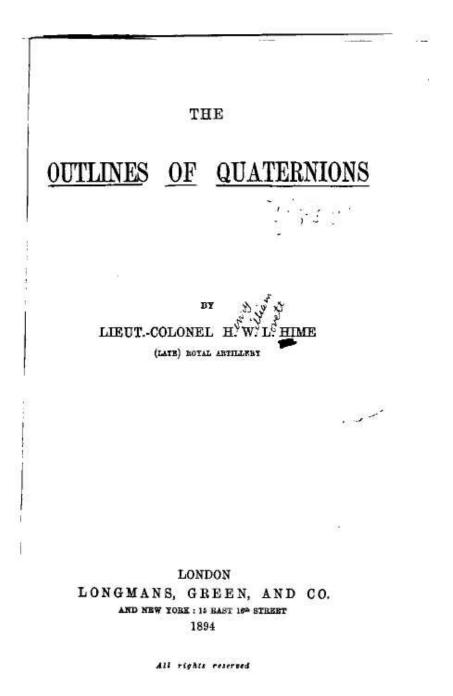
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H. W. L. HIME

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