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J. W. HOWARD

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tinue to stand with no evident sign of insecurity, though such a condition of things is directly in contradiction to the results given by the treatment.

A proper consideration of all the influences existing will not demonstrate the instability of a structure which stands with every evidence of security.

In the treatment which follows, the ring stones will be considered to be unchangeable in shape, or perfectly rigid. This hypothesis eliminates from the question all consideration of the condition of the ends of the arch at the skew-backs, and is not only admissible, but necessary, as the nearest approach to the actual condition of things. There is probably no arch in existence in which perfect continuity from skew-back to skew-back has been preserved after the removal of the centre. If the arch is assumed to be elastic, and fixed at the ends, this continuity must not only be absolutely perfect and extend through the skew-backs, but the whole structure must settle uniformly. If the ends are assumed to be hinged, continuity must hold between the skew-backs, and uniform settlement must take place. In consequence of the number of joints which are observed to open in many existing arches, even upon hasty examination, it is in the highest degree probable that neither perfect continuity nor perfectly uniform settlement occurs in any case.

It is, therefore, necessary to make the case one of pure statics only, instead of combined statics and strains.

It is supposed that no special devices, such as metallic clamps, are used to bind together the ring stones. With such appliances continuity and attendant conditions, might at least approximately be attained.

It is also to be assumed that no such devices are used to secure the ring stones to the spandril walls or filling. For the reasons already given, then, all forces acting on the arch ring must be directed inward, or in other words they must be pressures. The cement joint or joints between the ring stones and spandril can not be considered capable of resisting tension. It is not implied that good hydraulic cement mortar cannot resist tension. But it *is* true that if a mortar joint in an arch is capable of resisting any tension at all, it is impossible to determine to what extent it can be relied upon in that capacity; and in some joints, in good work, even contact does not exist. The real condition of things for a masonry arch will be more nearly described, therefore, by assuming that the arch ring is subject to *pressures* only. It then becomes necessary to determine just what these pressures are.

The vertical loading of an arch consists of two parts: the fixed, or permanent load and the moving load. The fixed load includes the spandril filling, or backing, on which the moving load and a part of the permanent load is applied, and which rests directly on the ring stones. The arch ring, therefore, must receive all pressure from and through the spandril.

The resultant pressure on the arch ring may be decomposed into two parts, one of which is the vertical loading, and the other is called the "conjugate" pressure. In many cases, perhaps the majority, this conjugate pressure is horizontal, but in others it is not. In Fig. 6, if the backing $B\ D\ C$ and $C\ E\ F$ is composed of dry earth, of which $B\ E$ is the trace of the plane upper surface, it is known from the theory of internal stress, or earth pressure, that the conjugate pressures h_1, h_2, h_3, etc., against the different ring stones will be parallel to that upper surface $B\ E$. The resultant pressure against any stone, as K, will be that force whose components are h_2, and the vertical load comprised between the two vertical lines v and v' drawn from the joints between K and the two adjacent stones, combined with the weight of the stone itself. In other words the vertical load is simply the weight resting upon the stone in question, and this is to be added to that of the stone itself. This weight is a fixed quantity depending upon the loading; but the conjugate pressure (as h_2) *may have any value between the conjugate pressure of earth loaded with its own weight only, and the abutting power of earth.* This is a principle of great importance and it will be seen to have a very potent influence on the determination of the line of resistance.

Any conjugate pressure (as h_2) on the posterior surface of a ring stone is simply *the conjugate earth pressure exerted on that portion of a vertical plane included between two conjugate planes drawn through the upper extremities (as a and b) of the two bounding joints.*

These methods of determining the components of the resultant force acting on any stone are very simple and very important. In the figure $a\ c$ is vertical and $b\ c$ parallel to $B\ E$, but the methods just given are in no manner dependent upon those directions.

If the backing $B\ D\ C$ and $C\ E\ F$, (Fig. 6, Plate I.) is of masonry or any other tenacious material, the direction of the conjugate pressure cannot be determined; it will depend upon the nature of the material, character of work, etc. It *may* be parallel to the conjugate direction $B\ E$, but it will probably not be.

These conjugate pressures may, therefore, in this case be assumed to have any directions whatever which will give a line of resistance most favorable to stability, since this line of resistance may be supposed to exist at the instant before rupture. It is implied, of course, that the obliquities of these pressures, in reference to the surfaces on which they act, must not exceed the limiting angles of friction, or the angles of repose, for those surfaces.

In regard to the magnitudes of these pressures, they may have any values which will cause their greatest intensities to lie between the limits of zero and the working resistance to compression of either the backing or ring stones. The upper limit of the intensity will be the least of the two working resistances. If it were a mere question of statics, aside from the consideration of a safety factor, "ultimate resistance" would be used instead of "working resistance."

Thus, *within certain limits*, there is a capacity of adjustment, requisite to the conditions of stability, of the pressures h between the ring stones and backing.

It should be observed at this point that, if the backing is of good masonry and well bonded to the ring stones, the line of resistance may be located anywhere, within safe limits, between the upper surface of the backing and the soffit of the arch. In such a case, questions in regard to the stability can hardly arise; it (stability) is secured almost *a priori*.

There have, then, thus far, been shown the methods for finding the vertical and other component of the resultant pressure on each stone of the arch ring; and it is to be carefully remembered that this resultant includes the weight of the stone itself. These resultant pressures are simply the external forces acting upon the individual stones; and they are completely known, for when the character of the pressures h_1, h_2, etc., is once determined, their centres are easily found by known means. The lines of action of the vertical loading and weight, for each stone, are always known.

Now, let *any* portion of the arch, as $C\ L\ M\ G$, be considered. If *all* the external forces acting on this portion were *completely* known, a trial line of resistance could at once be constructed. But even if the external forces acting on the stones are given, those which act in the joints $C\ G$ and $L\ M$ are still unknown, both in magnitude and line of action. It is known from the first principles of statics, and shown graphically by the force polygon, that if all the forces in a balanced system, except two, are known, *any two which will close the polygon will hold the system in equilibrium*, provided that the resultant moment of the whole system about any axis normal to its plane (in the arch all forces may be taken to lie in one plane,) is equal to zero. From these considerations it at once follows that any one point in each of the joints $L\ M$ and $C\ G$ may be assumed as the centres of resistance for those joints, or, what is the same thing, as the points of application of the resultant forces acting on those joints. Let a' and a'', Fig. 6, be those assumed points.

The magnitudes and directions of the forces acting in $L\ M$ and $C\ G$, or through a' and a'', still remain to be found, and *the direction of one of them may be assumed.* The direction of the one acting through a'' will be assumed parallel to $B\ E$ and it will be called T_0; the line representing it is removed to one side, for the purpose of avoiding complication in the diagram. By taking the moments of T_0 and the resultant external forces acting on the ring stones about the point a', the magnitude of T_0 can at once be found. The remaining force, R, acting in the joint $L\ M$ becomes immediately determinate, both in direction and magnitude, and can be found by the use of the force polygon. These operations will presently be carried through in detail, by means of equations and diagrams.

The two forces, T_0 and R, which have just been determined, are but one of an indefinite number of pairs which may be used to complete the equilibrium of the portion $C\ L\ M\ G$. The remaining portion of the problem is to determine which of all these pairs will really exist in any given case. The complete solution of this question is perhaps impossible, but an approach to it, at least, may be made. The force or thrust T_0 is either partially or wholly a passive force called into action by the resultant external forces acting on the ring stones, which play the part of active forces.

By the principle of least resistance this passive force T_0 must be the least possible, consistent with the equilibrium of the structure.

The absolutely least value of T_0 consistent with stability can scarcely be found, because the operation by which it is sought is a tentative one. When once the point a' and direction of T_0 are assumed (which assumptions have been shown to be consistent with equilibrium), however approximately, the least value of T_0 is not difficult to fix.

In Fig. 6, Plate I, let g be the centre of gravity of the portion of the arch $A\ v''\ L\ M\ G$ and its load, and let W represent the weight which acts through that point, while l_1, as shown, is the horizontal distance of the line of action of W from the centre of resistance a'. Let h represent any of the forces h_1, h_2, h_3, etc. (for the sake of generality these are considered not to be parallel), and let l be the normal distance from the line of action of h to a_1; then the sum of all the moments of the forces h about a' will be Σhl. By taking moments, therefore, about a' there will result:

$$T_0 y_1 = Wl_1 + \Sigma hl \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\therefore T_0 = \frac{Wl_1 + \Sigma hl}{y_1} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In Eq. (2) y_1 is the lever arm of the thrust T_0, or, the normal distance of its line of action from a', whatever may be the direction of T_0.

If the stability of position of any joint is secured, the centre of resistance must not depart farther from the centre of figure than a certain safe distance. Denoting the thickness, or depth of the springing joint $L\ M$ by t, the limiting distance of a' from the centre of $L\ M$ will be indicated by $q\ t$. The distance $M\ a'$ will be taken as $\frac{1}{2}t - q\ t = t(\frac{1}{2} - q)$. This assumption fixes the location of a', but leaves that of a'' movable within safe limits.

With the position of a' fixed, however much a'' may be moved in the joint $C\ G$, the lever arms l_1 and l will remain unchanged. Consequently, *the highest position possible of a'', which makes y_1, the greatest possible, makes, by Eq. 2, T_0 the least possible*, with a given set of values for the forces h. In fact, the form of Eq. 2 shows, in general, *that in order to have T_0 as small as possible, a'' should be as high as possible.* In general, also, a' should be as low (or as near to M) as possible in the joint $M\ L$; but, since a movement of a' in some cases may increase l and decrease l_1, or *vice versa*, it may be necessary to locate that point in some position higher than its lowest in such cases.

Such, with a given set of values of the pressures h, is the general method of determining the most probable (*i. e.* least) value of the thrust T_0 by aid of the principle of least resistance.

It has been observed that the pressures h (usually conjugate to the vertical) have possible values lying between certain limits, and, in the general case, may have varying possible directions. Eq. 2 shows that, if other things remain the same, T_0 will decrease as Σhl decreases. Consequently, by the principle of least resistance, *the pressures, h, should be so taken that their magnitudes and lever arms may be the least possible; or, analytically,* Σhl *should be the least possible.*

The preceding methods and observations are perfectly general, and apply to all cases whatever; but if the characteristics of a given case are more simple, their applications become correspondingly easy.

It very frequently happens that the entire loading of an arch, if reduced to a uniform mass of the same density as that of the ring stones, may be taken as limited by a plane upper surface, $B\ E$, without sensible error. A *sloping* plane upper surface, like that in Fig. 6, will first be considered. With such a loading it frequently happens that the pressures h are conjugate to the vertical and parallel to $B\ E$, in which case their direction is fixed and cannot change however much the magnitudes or intensities may vary. Under these circumstances, with a given location of a', the lever arms l are invariable. In seeking the least value of T_0 by Eq. 2, therefore, the magnitudes h, alone, are to be taken as the least possible when y_1 and l_1 are fixed.

If the upper surface $B\ E$, Fig. 6, or conjugate direction, becomes horizontal, the forces h are horizontal forces, and the lever arms l are simply the vertical distances from the action lines of those forces to the point a'; and as before, in seeking T_0, Σhl is to be made as small as possible by decreasing h only. In this restricted case, if the springing joint $L\ M$ is horizontal, the lever arms l are absolutely invariable. Consequently, while, as before, a'' is to be taken as high as possible in the joint $C\ G$, the point a' is *always* to be placed as near as possible to M, because l_1 will then have its least value, and y_1 will be independ-