NOTES ON RANKINE'S APPLIED MECHANICS

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RANKINE'S APPLIED MECHANICS.

BY

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INTRODUCTION.

The following pages are the result of putting in permanent form some of the matter which it has been found expedient or necessary to give by dictation to students in the Worcester Free Institute, who pursue for the first time, the study of Rankine's Applied Mechanics. The object in their publication is not to furnish a key, or provide a substitute for diligent study and careful thought on the part of the student, but rather to encourage him by giving such suggestions, solutions, and references as experience has shown that the average student requires; thus economizing time in the preparation of the lesson, and also giving the instructor opportunity to devote the time spent in the class room to recitations, and to the application of the principles and formulæ of the lesson, to practical problems.

To what may be strictly called notes on the "Applied Mechanics," I have added a brief explanation and illustration of the method of producing the reciprocal diagram of stresses, substantially taken from "Economics of Construction," by R. E. Bow, C. E. Also a separate treatise on strength of beams, and an investigation of a particular problem relating to seven bar parallel motions, known as "Peaucillier's Parallel Motion."

This work has been prepared from materials drawn from various sources, especially from notes given by Prof. Eustis, of The Lawrence Scientific School, to the class of '68.

I have also received assistance from George H. White, B. S., a graduate of the Free Institute, and have inserted on several articles of the Applied Mechanics, notes which are entirely his own work. I have endeavored to make proper reference to works from which quotations or extracts have been taken.

The blank pages at the end are intended to receive such supplementary notes as the instructor may find adapted to the needs and capacity of his class.

GEORGE I. ALDEN.

WORCESTER FREE INSTITUTE, Feb. 1st, 1877.

INTEGRATION.

The following integrals are of frequent occurrence, and are here given for future reference:

(A) $$\int d\,x\ \sqrt{a^2 - x^2}$$

In the general formula for integrating by parts, $\int u\,d\,v = u\,v - \int v\,.\,d\,u$, let $u = \sqrt{a^2 - x^2}$, and $d\,x = d\,v$; then $d\,u = -\dfrac{x\,d\,x}{\sqrt{a^2 - x^2}}$ and $x = v$

$$\therefore \int d\,x\ \sqrt{a^2 - x^2} = x\ \sqrt{a^2 - x^2} + \int \frac{x^2\,d\,x}{\sqrt{a^2 - x^2}}$$

Again,

$$\therefore \int d\,x\ \sqrt{a^2 - x^2} = \int \frac{d\,x\,(a^2 - x^2)}{\sqrt{a^2 - x^2}} = -\int \frac{x^2\,d\,x}{\sqrt{a^2 - x^2}} + \int \frac{a^2\,d\,x}{\sqrt{a^2 - x^2}}$$

$$\therefore 2\int d\,x\ \sqrt{a^2 - x^2} = x\,\sqrt{a^2 - x^2} + \int \frac{a^2\,d\,x}{\sqrt{a^2 - x^2}}$$

$$\int \frac{a^2\,d\,x}{\sqrt{a^2 - x^2}} = \int a^2 \frac{d\left(\frac{x}{a}\right)}{\sqrt{1 - \frac{x^2}{a^2}}} = a^2 \sin^{-1}\frac{x}{a} + C.$$

$$\therefore 2\int d\,x\ \sqrt{a^2 - x^2} = x\,\sqrt{a^2 - x^2} + a^2\ \sin^{-1}\frac{x}{a} + C$$

$$\therefore \int d\,x\ \sqrt{a^2 - x^2} = \frac{x}{2}\ \sqrt{a^2 - x^2} + \frac{a^2}{2}\ \sin^{-1}\frac{x}{a} + C.$$

(B) $$\int \frac{d\,x}{\sqrt{a^2 + x^2}}$$

Let $\sqrt{a^2 + x^2} = z - x$: Then $a^2 + x^2 = z^2 - 2z\,x + x^2$ or

$a^2 = z^2 - 2\,z\,x$. The differential of this equation is $0 = 2\,z\,d\,z - 2\,z\,d\,x - 2\,x\,d\,z \;\therefore\; d\,x = \frac{(z-x)\,d\,z}{z}$

$\therefore \int \frac{d\,x}{\sqrt{a^2 - x^2}} = \frac{(z-x)\,d\,z}{(z-x)\,z} = \int \frac{d\,z}{z} = \log_e z + C = \log_e (x + \sqrt{a^2 + x^2}) + C$

(C) $\int d\,x\,\sqrt{a^2 + x^2}$

In the formula for integration by parts, $\int u\,d\,v = u\,v - \int v\,d\,u$, let $\sqrt{a^2 + x^2} = u$ and $d\,x = d\,v$. Then $x = v$ and $\frac{x\,d\,x}{\sqrt{a^2 + x^2}} = d\,u$

$\therefore \int d\,x\,\sqrt{a^2 + x^2} = x\,\sqrt{a^2 + x^2} - \int \frac{x^2\,d\,x}{\sqrt{a^2 + x^2}}$. Again, $\int d\,x\,\sqrt{a^2 + x^2} = \int \frac{d\,x\,(a^2 + x^2)}{\sqrt{a^2 + x^2}} = \int \frac{a^2\,d\,x}{\sqrt{a^2 + x^2}} + \int \frac{x^2\,d\,x}{\sqrt{a^2 + x^2}}$

$\therefore 2 \int d\,x\,\sqrt{a^2 + x^2} = x\,\sqrt{a^2 + x^2} + a^2 \int \frac{d\,x}{\sqrt{a^2 + x^2}}$

By integral B, we have $a^2 \int \frac{d\,x}{\sqrt{a^2 + x^2}} = a^2 \log_e (x + \sqrt{a^2 + x^2}) + C$.

$\therefore \int d\,x\,\sqrt{a^2 + x^2} = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2} \log_e (x + \sqrt{a^2 + x^2}) + C.$

(D) $\int x^2\,d\,x\,\sqrt{a^2 - x^2}$

In the formula $\int u\,d\,v = u\,v - \int v\,d\,u$, let $x\,d\,x\,\sqrt{a^2 - x^2} = d\,v$, and $x = u$: then $v = -\frac{1}{3}(a^2 - x^2)^{\frac{3}{2}}$ and $d\,u = d\,x$

$\therefore \int x^2\,d\,x\,\sqrt{a^2 - x^2} = -\frac{x}{3}(a^2 - x^2)^{\frac{3}{2}} + \frac{1}{3}\int d\,x\,(a^2 - x^2)^{\frac{3}{2}}$

$$= -\frac{x}{3}(a^2 - x^2)^{\frac{3}{2}} + \frac{1}{3}\int d\,x\,(a^2 - x^2)\,\sqrt{a^2 - x^2}$$

$$= -\frac{x}{3}(a^2 - x^2)^{\frac{3}{2}} + \frac{1}{3}\int dx\, a^2 \sqrt{a^2 - x^2}$$

$$- \frac{1}{3}\int x^2\, dx\, \sqrt{a^2 - x^2}$$

Transposing the last term of the last member, we have

$$\frac{4}{3}\int x^2\, dx\, \sqrt{a^2 - x^2} = -\frac{x}{3}(a^2 - x^2)^{\frac{3}{2}} + \frac{1}{3}\int a^2\, dx \sqrt{a^2 - x^2}$$

From integral A we have

$$\frac{1}{3}a^2\int dx\, \sqrt{a^2 - x^2} = \frac{a^2}{3}\left\{\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\} + C.$$

$$\therefore \frac{4}{3}\int x^2\, dx\, \sqrt{a^2 - x^2} = -\frac{x}{3}(a^2 - x^2)^{\frac{3}{2}} + \frac{a^2 x}{6}\sqrt{a^2 - x^2}$$

$$+ \frac{a^4}{6}\sin^{-1}\frac{x}{a} + C'.$$

$$\therefore \int x^2\, dx\, \sqrt{a^2 - x^2} = -\frac{x}{4}(a^2 - x^2)^{\frac{3}{2}} + \frac{a^2 x}{8}\sqrt{a^2 - x^2}$$

$$+ \frac{a^4}{8}\sin^{-1}\frac{x}{a} + C.$$

This integral, taken between the limits a and 0, reduces to $\frac{a^4}{8}\sin^{-1}\frac{x}{a}$, which can easily be memorized.

(E) $$\int \cos^2\theta\, d\theta.$$

Integrals of the powers of $\sin\theta$ and $\cos\theta$ are found by substituting for these functions of θ, their values in terms of the multiple angles, as in the following case:

$$\int \cos^2\theta\, d\theta = \int \frac{1 + \cos 2\theta}{2}\, d\theta = \frac{\theta}{2} + \frac{\sin 2\theta}{4} + C.$$

Article 83.*

In solving the following problems the student should always sketch a figure representing the surface or solid under consideration, and one of the elementary parts into which it is conceived to be divided, and determine the limits for the integral by an inspection of this figure. In double or triple integrations, we

Note.—The first three integrals are taken from Todhunter's Integral Calculus.
Note.—The abbreviation A. M. will be used for Rankine's Applied Mechanics.
* The reference is to Article 83 *of the Applied Mechanics*.

must, in general, first integrate with respect to one of the variables, *between the proper limits*, and express the result in terms of the other variables. This may be continued until the complete integral is obtained.

To illustrate this process take the general formulæ for center of gravity of a solid, viz:

$$x_0 = \frac{\int\int\int x\,dx\,dy\,dz}{\int\int\int dx\,dy\,dz}$$ and similar values for y_0 and z_0.

The following application of these formulæ is found in Todhunter's Analytical Statics.

Problem. Find the center of gravity of the eighth part of an ellipsoid cut off by the three principal planes.

Let Fig. (1) represent the solid in question, the equation of the surface being $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$ (1)

If we integrate first with respect to z, between the limits z_1 and zero, we include all the elements $(dx\,dy\,dz)$ in the prism P Q. Next integrate with respect to y between the limits y_1 and zero. We thus include all the prisms in the slice between the planes l L and m M.

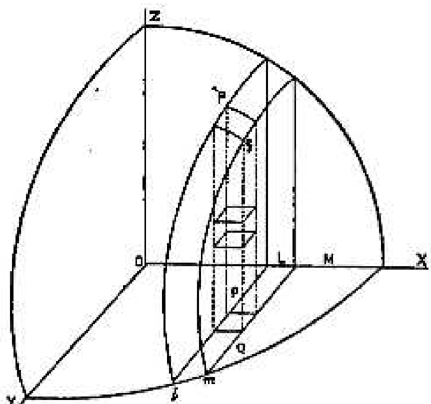

Fig. 1.

From Equation (1), $z_1 = c\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}$; from Equation of ellipse in the plane X Y, $y_1 = b\sqrt{1 - \frac{x^2}{a^2}}$