

**CURVE TRACING IN  
CARTESIAN  
COORDINATES, PP. 1-83**

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Curve Tracing in Cartesian Coordinates, pp. 1-83 by William Woolsey Johnson

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CURVE TRACING

IN

CARTESIAN COORDINATES

BY

WILLIAM WOOLSEY JOHNSON

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## PREFACE

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THIS book relates, not to the general theory of curves, but to the definite problem of ascertaining the form of a curve given by its equation in Cartesian coordinates, in such cases as are likely to arise in the actual applications of Analytical Geometry. The methods employed are exclusively algebraic, no knowledge of the Differential Calculus on the part of the reader being assumed.

I have endeavored to make the treatment of the subject thus restricted complete in all essential points, without exceeding such limits as its importance would seem to justify. This it has seemed to me possible to do by introducing at an early stage the device of the Analytical Triangle, and using it in connection with all the methods of approximation.

In constructing the triangle, which is essentially Newton's parallelogram, I have adopted Cramer's method of representing the possible terms by points, with a distinguishing mark to indicate the actual presence of the term in the equation. These points were regarded by Cramer as marking the centres of the squares in which, in New-



ton's parallelogram, the values of the terms were to be inscribed; but I have followed the usual practice, first suggested, I believe, by Frost, of regarding them merely as points referred to the sides of the triangle as coordinate axes. It has, however, been thought best to return to Newton's arrangement, in which these analytical axes are in the usual position of coordinate axes, instead of placing the third side of the triangle, like De Gua and Cramer, in a horizontal position.

The third side of the Analytical Triangle bears the same relation to the geometrical conception of the line at infinity that the other sides bear to the coordinate axes. I have aimed to bring out this connection in such a way that the student who desires to take up the general theory of curves may gain a clear view of this conception, and be prepared to pass readily from the Cartesian system of coordinates, in which one of the fundamental lines is the line at infinity, to the generalized system, in which all three fundamental lines are taken at pleasure.

Lists of examples for practice will be found at the end of each section. These examples have been selected from various sources, and classified in accordance with the subjects of the several sections.

W. W. J.

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