FUNDAMENTAL CONCEPTIONS OF MODERN MATHEMATICS: VARIABLES AND QUANTITIES

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649088126

Fundamental conceptions of modern mathematics: Variables and Quantities by Robert P. Richardson & Edward H. Landis

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

www.triestepublishing.com

ROBERT P. RICHARDSON & EDWARD H. LANDIS

FUNDAMENTAL CONCEPTIONS OF MODERN MATHEMATICS: VARIABLES AND QUANTITIES

Trieste

BY THE SAME AUTHORS

NUMBERS, VARIABLES, AND MR. RUSSELL'S PHILOSOPHY
(REPRINTED FROM "THE MONIST" OF JULY, 1915)

A PAMPHLET OF 50 PAGES

THE OPEN COURT PUBLISHING CO., CHICAGO AND LONDON

(Price, 10 Cents)

FUNDAMENTAL CONCEPTIONS

OF

MODERN MATHEMATICS

Variables and Quantities

WITH A DISCUSSION OF THE GENERAL CONCEPTION OF FUNCTIONAL RELATION

BY

ROBERT P. RICHARDSON

AND

EDWARD H. LANDIS

CHICAGO AND LONDON
THE OPEN COURT PUBLISHING COMPANY
1916

PREFACE.

The present work is essentially one of constructive criticism. It is, we believe, the first attempt made on any extensive scale to examine critically the fundamental conceptions of Mathematics as embodied in the current definitions. The purpose of our examination is not solely or even chiefly to show the presence of error, but to promote the development of a more scientific doctrine. In expounding our own views we have often been obliged to find fault with those of others: but we have not gone out of our way for the sake of mere criticism; we have merely cleared away false doctrine preparatory to replacing it with true. Our work, though in a sense dealing with definitions, does not have as its essential scope questions as to the words to be used in expressing something about which there is universal agreement; it really deals with the conceptions underlying the definitions where there is, as will be shown, a great diversity of view. Further than a discussion of definitions (in this sense) we do not go, and though we have at times occasion to enunciate axioms and theorems we never set down a demonstration. It is indeed undeniable that a discipline consisting of definitions alone would be perfectly futile, but this is no argument against deeming the definitions of a science worthy of a separate exposition. How far from being systematic is the treatment of the definitions of Mathematics in most mathematical writings will be appreciated by all who have given their attention to the matter. Definitions are laid down only as they are needed for the work in hand, and in their formulation attention is given, not to the needs of mathematical science as a whole, but to those of a single book—too often a book whose sole purpose is to enable more or less stupid youths to pose as graduates of a course in Mathematics. As to the articles of original research published in mathematical journals, definitions are hardly to be found in them at all. This state of affairs has reacted upon the demonstrations of Mathematics. When a systematic nomenclature and its concomitant, a clear and connected view of matters, are lacking, precision in statement cannot be expected. Nor is it to be found, and by far the most difficult task to the reader of a work on advanced Mathematics is not appreciating the cogency of the reasoning employed—or depreciating it, as one is sometimes compelled to do—but ascertaining what the author really means. This in no small number of cases is something very different from what he has said. Such a state of affairs does not rule in elementary Geometry; due in large measure to the Euclidean custom of beginning a demonstration with a precise statement of the fact about to be proven; this in turn necessitating more attention to matters of definition than modern mathematicians have thought fit to give. Mathematics to-day is indeed far behind most other sciences as regards lucidity of exposition. In a comparatively short time a young man of average ability can become so far familiar with Chemistry or Botany or Zoölogy, as to be able to read intelligently a work in any department of the science whatsoever. But this is not the case with Mathematics—a student far above mediocrity, who has taken the best University Course in Mathematics to be found, will come across mathematical works as unintelligible to him as Chinese or Choctaw. It is not merely that he finds himself unfamiliar with the theorems proven in such works: this would be neither surprising nor detrimental; but he will not even be able to understand what it is that the theorems are about. And to gain the knowledge requisite for this will not be a matter of consulting a lexicon; but one of hard study for several months.[1] This state of affairs is not, we hold, an unavoidable one due to the peculiar difficulties of Mathematics. It is due to the lack of systemization; and in particular to the failure of text-books to give any thorough exposition of the fundamental conceptions of Mathematics. The thirst for so-called "original research," and the credit attached to it, has led mathematicians to disregard such matters. The investigation, for example, of some particular differential equation not yet touched upon is classed as "original" work, while investigation of the current doctrine of differentiation is not. And by implication the impression is conveyed that work of the former type requires a higher degree of intellect than the latter—an impression very far from the truth. Thus the one is encouraged, the other discouraged; and in many quarters the impression prevails that there is nothing more to be done at the foundations of Mathematics; that the only object of a mathematician should be to rear

[1] As an illustration of the difficulties in the way of acquiring a thorough knowledge of a branch of mathematics, we may mention that Hamilton, assuredly no tyro in Vector Analysis, found the *Ausdehnungslehre* so obscure that he avowed himself unable to understand Grassmann's system in all its details. And Herschel, in turn, after reading three chapters of Hamilton's *Lectures on Quaternions*, was obliged "to give up in despair" his hope of mastering the subject. This was some years ago, but what change has since taken place in methods of mathematical exposition has not been a change for the better.

the superstructure still higher, leaving the old foundations alone. In fact, however, the great desideratum in Mathematics at the present day is, a rebuilding of the foundations, and a readjustment and systemization of what has been built upon them. There is needed a scientific exposition of the definitions, and a complete enumeration, with specific enunciation, of the axioms and postulates. After this (but not before) should come a systematic statement of the theorems, the conditions under which each is valid being stated with perfect precision. It is of little avail to have the theorem of some "original investigator" hidden away in a back number of some mathematical journal, and even there loosely stated or (as is more commonly the case) not explicitly stated at all.

This much-needed revision of Mathematics ought undoubtedly to be made from a philosophical standpoint, there being constantly maintained rigid adherence to the requirements of a sane Metaphysics in the best sense of the word and to the canons of a sound Logic. It is quite clear that unless our fundamental conceptions and principles accord with the one, and our processes of deduction with the other, we cannot develop anything worthy of the name of a deductive science. Unfortunately too many mathematicians look askance upon the application of philosophical doctrine to Mathematics. With but few exceptions, authors of mathematical works and teachers of the subject cultivate Mathematics as an art. They often show extraordinary ingenuity in the solution of problems and in the transformation of formulas, while giving little heed to the realities represented by their symbols and the processes of inference corresponding to their symbolic transformations. Were this all, no objection could be raised by those who wish to see Mathematics developed as a science. The bricklayer and carpenter are useful members of society, even though ignorant of the science of Mechanics. But too often the conventional mathematician arrogantly assumes, toward the philosophical side of the question, an attitude like that of the illiterate artisan toward physical science. He stigmatizes any attempt at logical precision as of no practical value; and is indeed in one respect worse than the carpenter or bricklayer, since the latter makes no claim to the title of scientist, while the artisan mathematician would arrogate this to himself to the exclusion of the philosophical investigator. Such an attitude is amusing, when one considers of how little bread-and-butter utility are many departments of Mathematics which find no lack of devotees. It is really remarkable how narrow many mathematicians are, not merely in their lack of knowledge, but in their ignorance of their own limitations. They are aware of these limitations only so far as the physical sciences are concerned. None of them would, for instance, venture to speak on a question of Botany without having studied the subject, and likewise a botanist who had never mastered the first book of Euclid would not dare to affirm it to be possible to square the circle; but a mathematician who has never even opened a book on Logic will calmly make a pronunciamento on logical doctrine as absurd as the paradoxes of modern circle squarers or the vagaries of the ignorant theologians who "refute" the theory of evolution. More excusable are those mathematicians who openly acknowledge their incompetence in the logical field; there is so much charlatanism, in Logic as well as in Metaphysics, that a person who has only seen certain works (not the least renowned) on philosophical matters, may be pardoned for giving up the whole subject in disgust. The *Logic* of Hegel, for example, has no more to do with the science of Logic