INVENTIONAL GEOMETRY: A SERIES OF QUESTIONS, PROBLEMS, AND EXPLANATIONS

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W. G. SPENCER

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Trieste

INVENTIONAL GEOMETRY.

INVENTIONAL GEOMETRY:

A Series

OF

QUESTIONS, PROBLEMS, AND EXPLANATIONS,

INTENDED TO

FAMILIARIZE THE PUPIL WITH GEOMETRICAL CONCEPTIONS,

TO

EXERCISE HIS INVENTIVE FACULTY,

AND

PREPARE HIM FOR EUCLID AND THE HIGHER MATHEMATICS.

BY

W. G. SPENCER.

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1860.

NOTE.

The first three-fourths of the Inventional Geometry originally appeared in Nos. 18, 19, 20, 21, 22, and 23 of the *Educator*. To this already published portion, now re-issued, is added another fourth, commencing at Question 343.

Should the system receive sufficient encouragement, the Author may be induced to publish an Illustrated Key, which will afford an opportunity to the Pupil of comparing his own solutions with those invented by others.

In the mean time the Author will be glad to receive from Teachers who try the influence of the Inventional Geometry upon their Pupils, any suggestions they may think of importance.

Blank books, suitable for entering the questions and their solutions, are kept on hand by the Publisher. Price, per dozen, from 9s. to 10s. according to quality.

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INVENTIONAL GEOMETRY.

INTRODUCTION.

When it is considered that by geometry the architect constructs our buildings, the civil engineer our railways; that by a higher kind of geometry, the surveyor makes a map of a county or of a kingdom; that a geometry still higher is the foundation of the noble science of the astronomer, who by it not only determines the diameter of the globe he lives upon, but as well the sizes of the sun, moon, and planets, and their distances from us and from each other; when it is considered also, that by this higher kind of geometry, with the assistance of a chart and a mariner's compass, the sailor navigates the ocean with success, and thus brings all nations into amicable intercourse,—it will surely be allowed that its elements should be as accessible as possible.

Geometry may be divided into two parts—practical and theoretical: the practical bearing a similar relation to the theoretical that arithmetic does to algebra. And just as arithmetic is made to precede algebra, should practical geometry be made to precede theoretical geometry.

Arithmetic is not undervalued because it is inferior to algebra, nor ought practical geometry to be despised because theoretical geometry is the nobler of the two.

However excellent arithmetic may be as an instrument for strengthening the intellectual powers, geometry is far more so; for as it is easier to see the relation of surface to surface and of line to line, than of one number to another, so it is easier to induce a habit of reasoning by means of geometry than it is by means of arithmetic. If taught judiciously, the collateral advantages of practical geometry are not inconsiderable. Besides introducing to our notice, in their proper order, many of the terms of the physical sciences, it offers the most favourable means of comprehending those terms, and impressing them upon the memory. It educates the hand to dexterity and neatness, the eye to accuracy of perception, and the judgment to the appreciation of beautiful forms. These advantages alone claim for it a place in the education of all, not excepting that of women. Had practical geometry been taught as arithmetic is taught, its value would scarcely have required insisting on. But the didactic method hitherto used in teaching it does not exhibit its powers to advantage.

Any true geometrician who will teach practical geometry by definitions and questions thereon, will find that he can thus create a far greater interest in the science than he can by the usual course; and on adhering to the plan, he will perceive that it brings into earlier activity that highly valuable but much neglected power, the power to invent. It is this fact that has induced the author to choose as a suitable name for it, the inventional method of teaching practical geometry.

He has diligently watched its effects on both sexes, and his experience enables him to say, that its tendency is to lead the pupil to rely on his own resources, to systematise his discoveries in order that he may use them, and to gradually induce such a degree of self-reliance as enables him to prosecute his subsequent studies with satisfaction: especially if they should happen to be such studies as Euclid's Elements, the use of the globes, or perspective.

A word or two as to using the definitions and questions. Whether they relate to the mensuration of solids, of surfaces, or of lines; whether they belong to common square measure, or to duodecimals; or whether they appertain to the canon of trigonometry; it is not the author's intention that the definitions should be learnt by rote; but he recommends that the pupil should give an appropriate illustration of each as a proof that he understands it.

Again, instead of dictating to the pupil how to construct a geometrical figure—say a square—and letting him rest satisfied with being able to construct one from that dictation, the author has so organized these questions that by doing justice to each in its turn, the pupil finds that when he comes to it, he can construct a square without aid.

The greater part of the questions accompanying the definitions require for their answers geometrical figures and diagrams, accurately constructed by means of a pair of compasses, a scale of equal parts, and a protractor, whilst others require a verbal answer merely. In order to place the pupil as much as possible in the state in which Nature places him, some questions have been asked that involve an impossibility.

Whenever a departure from the scientific order of the questions occurs, such departure has been preferred for the sake of allowing time for the pupil to solve some difficult problem; inasmuch as it tends far more to the formation of a self-reliant character, that the pupil should be allowed time to solve such difficult problem, than that he should be either hurried or assisted.

The inventive power grows best in the sunshine of encouragement. Its first shoots are tender. Upbraiding a pupil with his want of skill, acts like a frost upon them, and materially checks their growth. It is partly on account of the dormant state in which the inventive power is found in most persons, and partly that very young beginners may not feel intimidated, that the introductory questions have been made so very simple.

TO THE PUPIL.

When it is found desirable to save time, omit copying the definitions; but when time can be spared, copy them into the trial book, to impress the terms on the memory.

In constructing a figure that you know, use arcs if you prefer them; but in all your attempts to solve a problem, prefer whole circles to arcs. Circles are suggestive, arcs are not.

Always have a reason for the method you adopt, although you may not be able to express it satisfactorily to another. Such, for example, as this. If from one end of a line, as a centre, I describe a circle of a certain size, and then from the other end of the line, as another centre, I describe another circle of the same size, the points where those circles intersect each other, if they intersect at all, must have the same relation to one end of such line which they have to the other.

The most improving method of entering the solutions is to show, in a first figure, all the circles in full by which you have arrived at the solution, and to draw a second figure in ink, without the circles.

It is not so much the problems which you are assisted in performing, as the problems you perform yourself, that will improve your talents and benefit your character. Refrain, then, from looking at the constructions invented by other persons—at least till you have discovered a construction of your own. The less assistance you seek the less you will require, and the less you will desire.

As the power to invent is ever varying in the same person, and as no two persons have that power equally, it is better not to be anxious about keeping pace with others. Indeed, all your efforts should be free from anxiety. Pleasurable efforts are the most effective. Be assured that no effort is lost, though at the time it may appear so. You may improve more while studying one problem that is rather intricate to you, than while performing several that are easy. Dwell upon what the immortal Newton said of his own habit of study. "I keep," says he, "the subject constantly before me, and wait till the first dawnings open by little and little into a full and clear light."

DEFINITIONS AND QUESTIONS.

The science of relative quantity, solid, superficial, and linear, is called Geometry, and the practical application of it, Mensuration. Thus we have mensuration of solids, mensuration of surfaces, and mensuration of lines; and to ascertain these quantities it is requisite that we should have dimensions.

The top, bottom, and sides of a solid body, as a cube,* are called its faces or surfaces,† and the edges of these surfaces are called lines.

* The most convenient form for illustration is that of the cubic inch, which is a solid, having equal rectangular surfaces.

† A surface is sometimes called a superficies.

The distance between the top and bottom of the cube is a dimension called the height, depth, or thickness of the cube; the distance between the left face and the right face is another dimension, called the breadth or width; and the distance between the front face and the back face is the third dimension, called the length of the cube.

Thus a cube is called a magnitude of three dimensions.

The three terms most commonly applied to the dimensions of a cube are length, breadth, and thickness.

1. Place a cube with one face flat on a table, and with another face towards you, and say which dimension you consider to be the thickness, which the breadth, and which the length.

2. Show to what objects the word *height* is more appropriate, and to what objects the word *depth*, and to what the word *thickness*.

As a surface has no thickness, it has two dimensions only, length and breadth. Thus a surface is called a magnitude of two dimensions.

3. Show how many faces a cube has.*

When a surface is such, that a line placed anywhere upon it will rest wholly on that surface, such surface is said to be a plane surface.†

As a line has neither breadth nor thickness, it has one dimension only, that of length.

Thus a line is called a magnitude of one dimension.

4. Count how many lines are formed on a cube by the intersection of its six plane surfaces.

If that which has neither breadth, nor thickness, but length only, can be said to have any form, then a line is such, that if it were turned upon its extremities, each part of it would keep its own place in space.

We cannot with a pencil make a line on paper—we represent a line.

The boundaries or ends of a line are called points, and the intersection of two lines gives a point.

As a point has neither length, breadth, nor thickness, it is said to have no dimension. It has position only.

A point is therefore not a magnitude.

5. Name the number of points that are made by the intersection of the twelve lines of a cube.

We cannot with a pencil make a point on paper—we represent a point.

When any two straight lines meet together from any other two directions than those which are perfectly opposite, they are said to make an angle.

And the point where they meet is called the angular point.

Thus two lines that meet each other on a cube make an angle.

* The surfaces of a cube are considered to be plane surfaces.

† When the word *line* is used in these definitions and questions a straight line is always meant.