ESSAYS ON THE THEORY OF NUMBERS

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Essays on the Theory of Numbers by Richard Dedekind

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RICHARD DEDEKIND

ESSAYS ON THE THEORY OF NUMBERS

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ESSAYS

ON THE

THEORY OF NUMBERS

I. CONTINUITY AND IRRATIONAL NUMBERS
II. THE NATURE AND MEANING OF NUMBERS

BY

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CONTINUITY AND IRRATIONAL NUMBERS

CONTINUITY AND IRRATIONAL NUMBERS.

MY attention was first directed toward the considerations which form the subject of this pamphlet in the autumn of 1858. As professor in the Polytechnic School in Zürich I found myself for the first time obliged to lecture upon the elements of the differential calculus and felt more keenly than ever before the lack of a really scientific foundation for arithmetic. In discussing the notion of the approach of a variable magnitude to a fixed limiting value, and especially in proving the theorem that every magnitude which grows continually, but not beyond all limits, must certainly approach a limiting value, I had recourse to geometric evidences. Even now such resort to geometric intuition in a first presentation of the differential calculus, I regard as exceedingly useful, from the didactic standpoint, and indeed indispensable, if one does not wish to lose too much time. But that this form of introduction into the differential calculus can make no claim to being scientific, no one will deny. For myself this feeling of dissatisfaction was so overpowering that I made the fixed resolve to keep meditating on the question till I should find a