AN ELEMENTARY TREATISE ON SPHERICAL ASTRONOMY: ADAPTED TO A COURSE OF INSTRUCTION IN CIVIL ENGINEERING

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AN ELEMENTARY TREATISE ON SPHERICAL ASTRONOMY: ADAPTED TO A COURSE OF INSTRUCTION IN CIVIL ENGINEERING

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AN

ELEMENTARY TREATISE

ON

SPHERICAL ASTRONOMY:

ADAPTED TO A

COURSE OF INSTRUCTION

IN

CIVIL ENGINEERING.

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PREFACE.

The following course in Spherical Astronomy has been prepared for the use of the Author's classes, and is intended to include those applications of astronomy which fall within the province of the Civil Engineer. It assumes a preliminary knowledge of general Descriptive Astronomy, and is also designed to be supplemented by a course of instruction and practice in the adjustment and use of portable astronomical instruments, and in practical computations. It is for this reason that theoretical solutions only have been given of the various problems considered, all examples having been omitted; but the development of the working formulæ has been carried to the point required for their practical application, and the results are given in a form adapted to immediate use.

CONTENTS.

SPHERICAL ASTRONOMY.

CHAPTER I.

SPHERICAL PROBLEMS.

1. In *Spherical Astronomy* the real *distances* and *magnitudes* of the celestial bodies are not considered, but only their relative *directions*. Hence, whatever may be their actual distances from the observer, they are all regarded as situated on the surface of a *Celestial Sphere* of indefinitely great radius, of which the earth is the center.

2. The fundamental definitions of Astronomy are illustrated in Fig. 1, which represents the principal circles of the celestial sphere projected on the plane of the meridian.

The observer being supposed to be in north latitude, HZR is the *meridian*, HAR the *horizon*, ZA the *prime vertical*, EQ the *equator*, CD the *ecliptic*, V the *vernal equinox*, Z the *zenith*, P the *north pole*, H the *north point*, R the *south point*, S the *place of a star*, ZO the star's *vertical circle*, PM its *hour circle*, and SL its *circle of latitude*.

3. The co-ordinates which determine the position of a celestial body and that of the observer, are reoresented by the following notation:

$ZE = \phi =$ latitude of the place,
$PZ = \psi =$ colatitude of do.,
$SO = h =$ star's altitude,
$ZS = z =$ " zenith distance,
$PZS = HO = Z =$ " azimuth from north point,
$SZR = OR = Z' =$ " " from south point,
$ZPS = EM = P =$ " hour angle,
$ZSP = S =$ " parallactic angle,
$AO = a =$ " amplitude,
$VM = \alpha =$ " right ascension,
$MS = \delta =$ " declination,
$PS = p =$ " polar distance,
$VL = L =$ " longitude,
$LS = \lambda =$ " latitude,
$CVE = \omega =$ obliquity of the ecliptic.

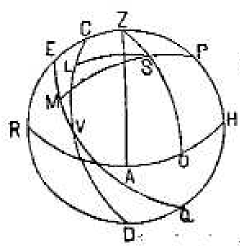

FIG. I.

4. Since PE, ZO, PM, HA and RA are quadrants, we have

$$\psi = 90^\circ - \phi \qquad (1)$$
$$z = 90^\circ - h \qquad (2)$$
$$p = 90^\circ - \delta \qquad (3)$$
$$a = 90^\circ - Z \qquad (4)$$
$$a = Z' - 90^\circ \qquad (5)$$
$$\text{whence } Z' = 180^\circ - Z \qquad (6)$$

5. Many of the most important problems of Spherical Astronomy can be reduced to the solution of the spherical triangle PZS, Fig. 2, formed by joining the pole, the zenith and the place of a star, by arcs of great circles.

The three sides of this triangle are

$PZ = 90° - \phi =$ colatitude,
$PS = 90° - \delta =$ star's polar distance,
$ZS = 90° - h =$ " zenith distance,

and the three angles are

$P =$ star's hour angle,
$Z =$ " azimuth from north point,
$S =$ " parallactic angle.

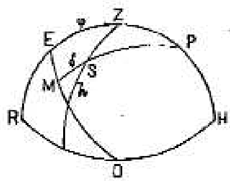

FIG. 2.

6. The following well-known formulæ of Spherical Trigonometry, applied to the triangle PZS, will furnish most of the general equations required in the discussions which follow. Denoting the angles of any spherical triangle by A, B, C, and its sides by a, b, c, we have

$$\left.\begin{aligned} \sin a \sin B &= \sin b \sin A \\ \sin b \sin C &= \sin c \sin B \\ \sin c \sin A &= \sin a \sin C \end{aligned}\right\} \quad (7)$$

$$\left.\begin{aligned} \cos a &= \cos b \cos c + \sin b \sin c \cos A \\ \cos b &= \cos c \cos a + \sin c \sin a \cos B \\ \cos c &= \cos a \cos b + \sin a \sin b \cos C \end{aligned}\right\} \quad (8)$$

$$\left.\begin{aligned} \sin a \cos B &= \sin c \cos b - \cos c \sin b \cos A \\ \sin b \cos C &= \sin a \cos c - \cos a \sin c \cos B \\ \sin c \cos A &= \sin b \cos a - \cos b \sin a \cos C \end{aligned}\right\} \quad (9)$$

$$\left.\begin{aligned} \sin^2 \tfrac{1}{2} A &= \frac{\sin (s - b) \sin (s - c)}{\sin b \sin c} \\ \sin^2 \tfrac{1}{2} B &= \frac{\sin (s - c) \sin (s - a)}{\sin c \sin a} \end{aligned}\right\} \quad (10)$$

in which $s = \tfrac{1}{2} (a + b + c)$.

7. If we apply formulæ (7), (8) and (9) to the triangle PZS, making $A = P$, $B = Z$, $C = S$, $a = 90° - h$, $b = 90° - \delta$, $c = 90° - \phi$, we shall obtain the following

General Astronomical Formulæ.

$$\cos h \sin Z = \cos \delta \sin P \tag{11}$$
$$\cos \delta \sin S = \cos \phi \sin Z \tag{12}$$
$$\cos \phi \sin P = \cos h \sin S \tag{13}$$

$$\sin h = \sin \delta \sin \phi + \cos \delta \cos \phi \cos P \tag{14}$$
$$\sin \delta = \sin \phi \sin h + \cos \phi \cos h \cos Z \tag{15}$$
$$\sin \phi = \sin h \sin \delta + \cos h \cos \delta \cos S \tag{16}$$

$$\cos h \cos Z = \sin \delta \cos \phi - \cos \delta \sin \phi \cos P \tag{17}$$
$$\cos \delta \cos S = \sin \phi \cos h - \cos \phi \sin h \cos Z \tag{18}$$
$$\cos \phi \cos P = \sin h \cos \delta - \cos h \sin \delta \cos S \tag{19}$$

By making the proper substitutions in these equations we may find the formulæ for a body in any position in the heavens.

8. Given the *latitude of the place* and the *declination of the body*, to find its *altitude* and *azimuth* when it is on the *six hour circle*.

In this position the hour angle $P = 6$ *hours* $= 90°$, hence $\sin P = 1$, $\cos P = 0$, and (14) becomes

$$\sin h = \sin \delta \sin \phi \tag{20}$$

(11) becomes $\cos h \sin Z = \cos \delta$

(17) becomes $\cos h \cos Z = \sin \delta \cos \phi$

whence by division,

$$\tan Z = \frac{\cot \delta}{\cos \phi} \tag{21}$$

Eqs. (20) and (21) are the expressions required.

9. Given the same data, to find the *hour angle* and *azimuth* of a body in the *horizon*.