LECTURES ON THE CALCULUS OF VARIATIONS. THE DECENNIAL PUBLICATIONS SECOND SERIES, VOL. XIV

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Alexander Ziwet

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BY

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PREFACE

THE principal steps in the progress of the Calculus of Variations during the last thirty years may be characterized as follows:

1. A critical revision of the foundations and demonstrations of the older theory of the first and second variation according to the modern requirements of rigor, by WEIERSTRASS, ERDMANN, DU BOIS-REYMOND, SCHEEFFER, SCHWARZ, and others. The result of this revision was: a sharper formulation of the problems, rigorous proofs for the first three necessary conditions, and a rigorous proof of the sufficiency of these conditions for what is now called a "weak" extremum.

2. WEIERSTRASS's extension of the theory of the first and second variation to the case where the curves under consideration are given in parameter-representation. This was an advance of great importance for all geometrical applications of the Calculus of Variations; for the older method implied —for geometrical problems—a rather artificial restriction.

3. WEIERSTRASS's discovery of the fourth necessary condition and his sufficiency proof for a so-called "strong" extremum, which gave for the first time a complete solution, at least for the simplest type of problems, by means of an entirely new method based upon what is now known as "WEIERSTRASS's construction."

These discoveries mark a turning-point in the history of the Calculus of Variations. Unfortunately they were given by WEIERSTRASS only in his lectures, and thus became known only very slowly to the general mathematical public. Chiefly under the influence of WEIERSTRASS's theory a vigorous activity in the Calculus of Variations has set in during the last few years, which has led—apart from extensions and simplifications of WEIERSTRASS's theory—to the following two essentially new developments:

4. KNESER's theory, which is based upon an extension of certain theorems on geodesics to extremals in general. This new method furnishes likewise a complete system of sufficient conditions and goes beyond WEIERSTRASS's theory, inasmuch as it covers also the case of variable end-points.

5. HILBERT's *a priori* existence proof for an extremum of a definite integral—a discovery of far-reaching importance, not only for the Calculus of Variations, but also for the theory of differential equations and the theory of functions.

To give a detailed account of this development was the object of a series of lectures which I delivered at the Colloquium held in connection with the summer meeting of the American Mathematical Society at Ithaca, N.Y., in August, 1901. And the present volume is, in substance, a reproduction of these lectures, with such additions and modifications as seemed to me desirable in order that the book could serve as a treatise on that part of the Calculus of Variations to which the discussion is here confined, viz., the case in which the function under the integral sign depends upon a plane curve and involves no higher derivatives than the first.

With this view I have throughout supplied the detail argumentation and introduced examples in illustration of the general principles. The emphasis lies entirely on the theoretical side: I have endeavored to give clear definitions of the fundamental concepts, sharp formulations of the problems, and rigorous demonstrations. Difficult points, such as the proof of the existence of a "field," the details in HILBERT's existence proof, etc., have received special attention.

For a rigorous treatment of the Calculus of Variations the principal theorems of the modern theory of functions of a real variable are indispensable; these I had therefore to presuppose, the more so as I deviate from WEIERSTRASS and KNESER in not assuming the function under the integral sign to be analytic. In order, however, to make the book accessible to a larger circle of readers, I have systematically given references to the following standard works: *Encyclopaedie der mathematischen Wissenschaften* (abbreviated *E.*), especially the articles on "Allgemeine Functionslehre" (PRINGSHEIM) and "Differential- und Integralrechnung" (VOSS); JORDAN, *Cours d'Analyse*, second edition (abbreviated *J.*); GENOCCHI-PEANO, *Differentialrechnung und Grundzüge der Integralrechnung*, translated by BOHLMANN and SCHEPP (abbreviated *P.*); occasionally also to DINI, *Theorie der Functionen einer veränderlichen reellen Grösse*, translated by LÜROTH and SCHEPP; STOLZ, *Grundzüge der Differential- und Integralrechnung*. The references are given for each theorem where it occurs for the first time; they may also be found by means of the index at the end of the book.

Certain developments have been given in smaller print in order to indicate, not that they are of minor importance, but that they may be passed over at a first reading and taken up only when referred to later on.

A few remarks are necessary concerning my attitude toward WEIERSTRASS's lectures. WEIERSTRASS's results and methods may at present be considered as generally known, partly through dissertations and other publications of his pupils, partly through KNESER's *Lehrbuch der Variationsrechnung* (Braunschweig, 1900), partly through sets of notes ("Ausarbeitungen") of which a great number are in circulation and copies of which are accessible to everyone in the library of the Mathematische Verein at Berlin, and in the Mathematische Lesezimmer at Göttingen.

Under these circumstances I have not hesitated to make use of WEIERSTRASS's lectures just as if they had been published in print.

My principal source of information concerning WEIERSTRASS's theory has been the course of lectures on the Calculus of Variations of the Summer Semester, 1879, which I had the good fortune to attend as a student in the University of Berlin. Besides, I have had at my disposal sets of notes of the courses of 1877 (by MR. G. SCHULZ) and of 1882 (a copy of the set of notes in the "Lesezimmer" at Göttingen for which I am indebted to PROFESSOR TANNER), a copy of a few pages of the course of 1872 (from notes taken by MR. OTT), and finally a set of notes (for which I am indebted to DR. J. C. FIELDS) of a course of lectures on the Calculus of Variations by PROFESSOR H. A. SCHWARZ (1898–99).

I regret very much that I have not been able to make use of the articles on the Calculus of Variations in the *Encyclopaedie der mathematischen Wissenschaften* by KNESER, ZERMELO, and HAHN. When these articles appeared, the printing of this volume was practically completed. For the same reason no reference could be made to HANCOCK's *Lectures on the Calculus of Variations.*

In concluding, I wish to express my thanks to PROFESSOR G. A. BLISS for valuable suggestions and criticisms, and to DR. H. E. JORDAN for his assistance in the revision of the proof-sheets.

OSKAR BOLZA.

THE UNIVERSITY OF CHICAGO,
August 28, 1904.

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