KEY TO HUNTER'S INTRODUCTION TO THE CONIC SECTIONS

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649252114

Key to hunter's introduction to the conic sections by John Hunter

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

www.triestepublishing.com

JOHN HUNTER

KEY TO HUNTER'S INTRODUCTION TO THE CONIC SECTIONS

Trieste

KEY

TO THE

'CONIC SECTIONS.'

KEY

TO

HUNTER'S INTRODUCTION

TO THE

CONIC SECTIONS.

BY THE

REV. JOHN HUNTER, M.A.

FORMERLY VICE-PRINCIPAL
OF THE TRAINING COLLEGE, BATTERSEA.

LONDON:
LONGMANS, GREEN, AND CO.
1866.

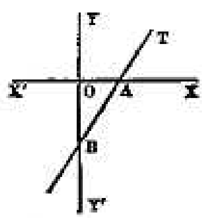
KEY

TO

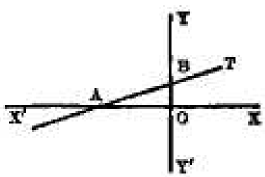
'CONIC SECTIONS.'

EXERCISES [A].

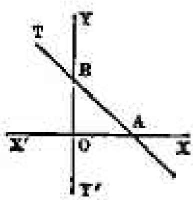
1. $3x-2y=4$. When $y=0$, then $x=1\frac{1}{3}$; and when $x=0$, then $y=-2$. Therefore, take $OA=1\frac{1}{3}$, $OB=2$. Then AB is the line required.



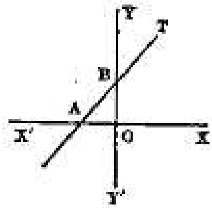
2. $x+3=4y$. When $y=0$, then $x=-3$; and when $x=0$, then $y=\frac{3}{4}$. Therefore, take $OA=3$, $OB=\frac{3}{4}$. Then AB is the line required.



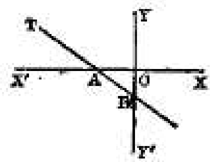
3. $4x+5y=6$. When $y=0$, then $x=1\frac{1}{2}$; and when $x=0$, then $y=1\frac{1}{5}$. Therefore, take $OA=1\frac{1}{2}$, $OB=1\frac{1}{5}$. Then AB is the line required.



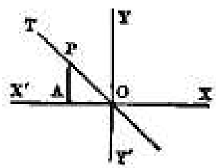
4. $\frac{1}{4}y-\frac{1}{3}x=5$; or $3y-4x=60$. When $y=0$, then $x=-15$; and when $x=0$, then $y=20$. Therefore, take $OA=15$, $OB=20$. Then AB is the line required.



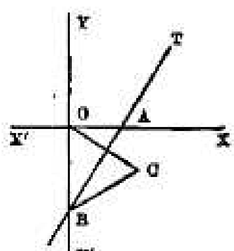
5. $3x+4y=-5$. When $y=0$, then $x=-1\frac{2}{3}$; and when $x=0$, then $y=-1\frac{1}{4}$. Therefore, take $OA=1\frac{2}{3}$, $OB=1\frac{1}{4}$. Then AB is the line required.



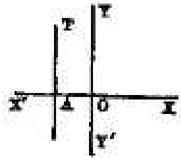
6. $x+y=0$; or $y=-x$. Here the coefficient of x is -1 which is the tangent of 135°. Therefore, make $TOX=135°$, and OT is the line required. For any point P being taken in OT will make $AP=OA$, or $y=-x$.



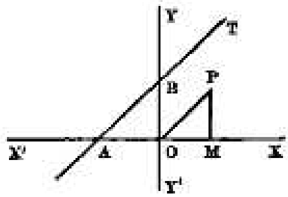
7. $x\sqrt{3}-y=8$; or $y=\sqrt{3}x-8$. Here the coefficient of x is $\sqrt{3}$, which is the tangent of 60°. Therefore, take $OB=8$, and on it describe the equilateral triangle OCB. The straight line BAT passing through the middle point of OC is the line required. For $TAX=OAB=90°-OBA=60°$.



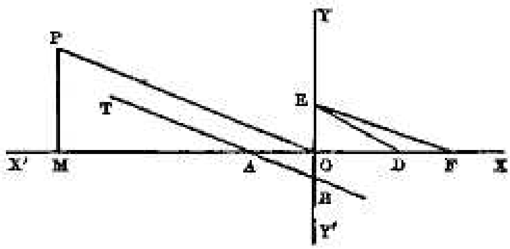
8. $x=-2$. This signifies that the abscissa of every point on the required line is $=-2$. Therefore, take $OA=2$, and the required line is AT, parallel to the axis of y.



9. $\frac{4}{9}x+\frac{2}{3}=\frac{1}{2}y$; or $y=\frac{8}{9}x+\frac{4}{3}$. Here the tangent of the angle which the line makes with the axis of x is $\frac{8}{9}$; therefore, take $\frac{\text{PM}}{\text{OM}}=\frac{8}{9}$, and join OP; then take OB$=1\frac{1}{3}$, and through B draw parallel to OP the required line AT.



10. $6y+x\sqrt{5}+\sqrt{10}=0$; or $y=-\frac{\sqrt{5}}{6}x-\frac{\sqrt{10}}{6}$.



Here the tangent being $-\frac{\sqrt{5}}{6}$, take OM$=6$, PM$=\sqrt{5}$, and join OP; then take OB$=\frac{1}{6}\sqrt{10}$, and through B draw parallel to OP the required line AT.

To find lines corresponding to $\sqrt{5}$ and $\sqrt{10}$, we may take OE$=1$, OD$=2$, and ED will be $=\sqrt{5}$; and if OF be taken $=3$, the line EF will be $=\sqrt{10}$.

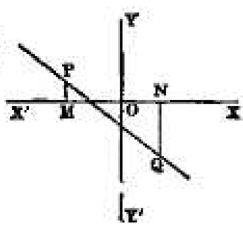
Exercises [B].

1. Take ON, QN, $=3$, 5; OM, PM, $=5$, 2; the line PQ is that required.

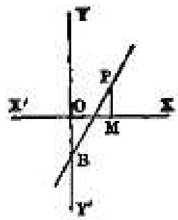
From art. 15 we have

$$y+5=\frac{2+5}{-5-3}(x-3)\text{; whence}$$

$$7x+8y=19.$$



2. Take OB=12; OM, PM, =11, 7; PB is the required line. Its equation (by art. **15**) is



$$y+12=\frac{7+12}{11+0}(x-0);$$

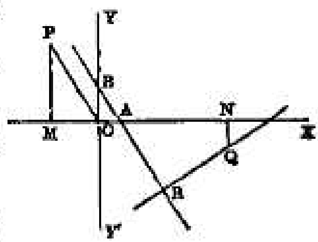
$$\text{or } 19x-11y=132.$$

3. The given line is $y=\frac{3}{7}x-\frac{36}{7}$. And, by art. **16**, the equation to a line passing through the point (0, 0) parallel to the given line is $y-0=\frac{3}{7}(x-0)$, or $3x=7y$.

Also the equation to a line passing through the point (13, 4) parallel to the given line is

$$y-4=\frac{3}{7}(x-13);\ \text{or } 3x-7y=11.$$

4. The given line is $y=-\frac{5}{3}x+\frac{4}{3}$. Take therefore $OB=\frac{4}{3}$, and $\frac{PM}{OM}=\frac{5}{3}$; join OP, and through B, parallel to OP, draw AB, which represents the given line.



Then take AN=5, QN=1, and the perpendicular QR on AB is the required line.

Now, by art. **18**, $y+1=\frac{3}{5}(x-5)$; or $3x-5y=20$.

5. The given line is $y=-\frac{7}{16}x+14$. And by art. **19** we have

$$y-30=\frac{-\frac{7}{16}\mp\sqrt{3}}{1\mp\frac{7}{16}\sqrt{3}}(x-5);$$

or, rationalising the denominators,

$$y-30=\frac{305\sqrt{3}+448}{\mp 109}(x-5);$$

which gives for the required equations

$$109y+(448\pm 305\sqrt{3})x-5(1102\pm 305\sqrt{3})=0.$$

6. By art. **20**, $PQ^2=(5+7)^2+(-4-12)^2=12^2+16^2$
$=400$; $\therefore$ $PQ=20$.

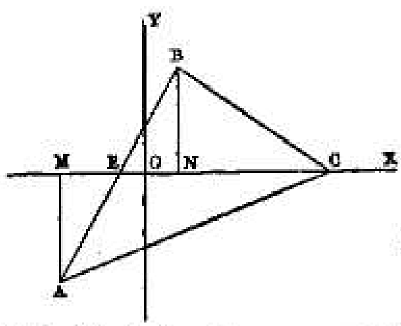
7. By art. **20**, $PQ^2=(-12-16)^2+(15-36)^2=28^2+21^2$
$=1225$; $\therefore$ $PQ=35$.

8. Here y may be taken as the common ordinate of the point of intersection. From the 1st and 2nd equations therefore we obtain $x=-16$, $y=-15$; which values being put for x and y in the 3rd equation form the identity $-45=-32-13$. Therefore the three lines meet at one point.

9. Taking y to denote the common ordinate of the point of intersection, we have $\frac{1}{2}x+3=6x-12$; whence we obtain $x=\frac{30}{11}$, $y=\frac{48}{11}$; and, by substitution, $\frac{48}{11}=-\frac{30}{11}m+8$; which gives $m=\frac{4}{3}$.

10. From the given equations we obtain $x=-5$, $y=4$, which values being made the coordinates of a point to which a line is drawn from the origin, the equation to that line will be $4x=-5y$, or $4x+5y=0$.

11. Take OM, AM, $=21, 25$; ON, BN, $=5, 26$; $OC=46$;



and join AB, BC, CA, to form the proposed triangle.

By similar triangles we have

$\frac{ME}{EN}=\frac{AM}{BN}$; or $\frac{ME}{MN}=\frac{AM}{AM+BN}$, that is, $\frac{ME}{26}=\frac{25}{51}$;

hence $ME=\frac{650}{51}$, and $EC=67-ME=\frac{2767}{51}$.

The required area is $=\frac{1}{2}EC(AM+BN)=2767\div 2=1383\frac{1}{2}$