ACCENTED FIVE-FIGURE LOGARITHMS OF NUMBERS FROM 1 TO 99 999 WITHOUT DIFFERENCES

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ARRANGED AND ACCENTED

BY

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PREFACE.

IN this volume the principle of accentuation is so applied to five-figure logarithms of Numbers as to enable calculations with them to yield results correct to five figures in Numbers, without using differences, proportional parts, or anti-logarithms.

Each logarithm is complete and separate, without any detached figures, so that the whole logarithm is seen at a glance, and this enables its cologarithm to be taken conveniently when required.

The accentuation exactly adapts every logarithm to a corresponding Number and to it alone, while using no extra figures; for instance, while the unaccented five-figure logarithms of 90591, 90592, and 90593 are identical, being ·95709, the accented logarithms are ·95709ʼ, ·95709, and ·95709.; the simple device of denoting mean values of 0,3, more or less, by a dot or a dash, fitting the logarithm to one Number only in any case. The doubt and hesitation and incorrectness of the ordinary mode is thus removed. The gain in exactitude of the system is clearly shown in the examples applied to numerical calculations of various sorts.

But this accentuation has also been designed with another object. It enables logarithms of trigonometrical ratios to yield more accurate results in angles. Thus, if we consider the greater portion of the quadrant, 0° to about 86°, to represent the mass of these very varying values, then the accented four-figure logarithms give results correct to minutes, the accented five-figure logarithms yield angles true to hundredths of a degree, and accented six-figure logarithms give results true to seconds; while with unaccented logarithms, five, six, and seven figures would be respectively required.

The trigonometrical portion of this book hence is applied to the centesimal division of the degree, a mode of great practical convenience for many purposes, adopted by some Continental scientific men and also by the American savants, who have greater facility than ourselves in utilising improved methods. In England, where minutes and seconds maintain their almost exclusive hold, this portion of the book may be less used, and will probably be applied mostly in surveying and engineering work, when five correct figures in result are sufficient, and rapidity is advantageous; but the logarithms of Numbers will become a necessity for all who calculate in five figures and wish to do so with the least labour and utmost convenience. The saving of figures, of time, and of risk of error in extensive calculations is large.

The triplet arrangement of lines of figures, both ways, secures immunity from one class of errors in reading, and the dispensing with proportional parts, practicable in most ordinary work with these tables, is a corresponding advantage.

The remarks made in the Preface and Introduction of my former work, "Accented Four-figure Logarithms and other Tables," also apply correspondingly to the use and accentuation of Five-figure Logarithms as far as regards general principles and methods. It will therefore be needless to repeat them here at full length.

L. D'A. J.

London, *December*, 1879.

INTRODUCTION.

THE ACCENTUATION OF LOGARITHMS.

THE common logarithm of a complete number is almost invariably expressed, as regards its mantissa, by a large number of figures whenever completion or full expression is desired. Thus, the logarithm of 15140, a Number perfectly complete in itself, may be expressed by—

3·180 125 875 164

and even then the logarithm is incomplete, as it should, to ensure completeness, be extended until all figures after the last significant digit become cyphers.

Taking, then, the mantissæ of logarithms generally to be interminable, the first point for consideration is, how many figures should be used in the mantissa? and the second, how can the tail or the remaining figures be best expressed?

It appears from the experience of the past that seven-figure logarithms alone were in practical use for ordinary purposes for a great number of years, but that latterly four-figure logarithms, originally used by the French, have been found sufficient to answer a great number of the purposes of ordinary calculation; the presumed advantage of such short logarithms being that about half the time spent on any calculation with seven-figure logarithms is thereby saved, also that the risk of error is diminished by half.

The reason that seven-figure and four-figure logarithms have thus been specially adopted, apart from the foregoing advantage, are that seven-figure logarithms enable seconds of arc to be obtained throughout the greater part of the quadrant, and that four-figure logarithms were supposed to enable calculations to give correct results to four figures in Numbers, a common accepted limit for the factors and results required in ordinary short calculations. It was also supposed that similarly five-figure logarithms could be employed to deal with numbers expressed in five figures, and yield results in five figures without important error.

For the higher purposes of the astronomer and of the scientific man, long logarithms are doubtless frequently necessary, as he requires results to hundredths of a second in some cases; but a great number of astronomical calculations can be done with four-figure logarithms; for the computers in the Nautical Almanac Office can calculate the whole of their occultations, almost all their eclipses, and all the places of the stars, excepting circumpolar stars, with four-figure trigonometrical logarithms, giving minutes of arc; they have not hitherto done so for want of such tables.

Again many astronomical calculations, such as the clearing of a lunar distance, may be so transformed in method as to admit of being done with minute tables. An example of this is given attached to "Accented Four-figure Logarithms."

The calculations of ordinary topographical survey, or of engineering survey, seldom require greater exactitude than 30 seconds, or perhaps 20 seconds; for much of their work four-figure logarithms to minutes would suffice; for the remainder, which certainly does not require exactitude to seconds, five-figure logarithms arranged to suit the centesimal division of the degree, that is, giving trigonometrical functions to every hundredth of a degree, would appear to be the most convenient and concise arrangement.

Lastly, for students who waste an enormous amount of time with their mistakes, and reduction of proportional parts and differences in seven-figure logarithms (still adhered to in schools), short logarithms, giving results by inspection either in four or in five figures, would be an important gain, representing months of educational time saved for better purposes.

Seven-figure logarithms are required for special purposes only, and when the whole of the seven figures can be utilised; their employment with differences for other purposes is waste of time and labour.

Next, as regards ordinary four-figure logarithms, and more especially the principle involved in attempting to use them with the object of getting results correct to four figures in Numbers. This has been shown in the examples attached to "Accented Four-figure Logarithms" to be fallacious. Practical men that attempt to use them in ordinary numerical calculations complain justly of the great defect, that in as many cases as not, the results are inaccurate; the fourth figure in Numbers being liable to be as much as 4 too much or too little, while whether it is so or not is a mere chance dependent on the balancing or non-balancing of small involved errors in the detail of the working of each example. The practical man, being told that in a long series of cases all such errors in results may balance each other on the whole, is not satisfied, and failing to obtain correct results in every case, while annoyed by the perpetual liability to error, looks on four-figure logarithms as a delusion.

An exactly corresponding liability to error exists in attempting to use five-figure logarithms for obtaining results correct to the fifth figure in Numbers; the attached examples demonstrate this clearly, and the list of errors following them, accompanied by those shown with seven-figure logarithms, should convince the most sceptical.

The remedy for this defect as regards four figures has been already given in accented four-figure logarithms, which enable the fourth figure in Numbers to be obtained with precision; while the corresponding remedy as regards five figures is offered in this book, which enables the fifth figure in Numbers to be obtained accurately. There are a few very special cases, such as those involving compound interest for a long term of years, and some others, not suited to five-figure computation at all, in which this accuracy cannot be expected: apart from those, the exactitude in the last figure is undoubted.

The cause of the defect, before referred to, is evidently the neglect of the tail, or rejected figures of the mantissa of the logarithm; and to investigate this neglect, the subject of augmentation and accentuation must be brought into consideration.

If the tail of the mantissa of a logarithm be simply cut off, and the logarithm be represented merely by the retained figures, as

3·180 125 875	by	3·18012
3·299 289 334	by	3·29928

and so forth; the utilised or retained portion is a low value of the logarithm; the error is thus left entirely in one direction, and its amount cannot exceed 1 in the last place of the retained figures.

But on adopting the principle of augmentation, which is in general use, the last of the retained figures is augmented by 1 whenever the first of the rejected figures is a full 5 or more than 5; thus the tail of the logarithm may or may not be partially expressed, as we represent

3·180 125 875	by	3·18013
and 3·180 134 763	by	3·18013

Under these circumstances the retained portion of the logarithm is nearer to exactitude than in the former case, as the error cannot be more than ½ of a unit in the last place; but it is always doubtful whether the error is plus or minus, and the limit of variation of the error still amounts to 1 as in the former case.

A further step towards exactitude is adopted by some writers through distinguishing an augmented from an unaugmented final figure by a mark thus,

3·180 1251	by	3·1801$\underline{3}$
3·180 1302	by	3·18013

the dash under the final figure indicating its real weakness. When this principle and method is adhered to, not only is the actual error less than ½ of a unit in the last figure, but the limit of variation of error is ½, instead of being 1. In other words the rejected unrepresented tail of the logarithm must under these circumstances be less than 0,5 while its mean value is 0,25, and the presence or absence of the mark of augmentation informs us whether it is plus 0,25 or minus 0,25. Thus the mean values of

3·1801$\underline{3}$ are 3·1801275
and 3·18013 and 3·1801325

and the former quantities indicate to us the latter values for use.

This method of *marked* augmentation is not common in England, where the former plan of *unmarked* augmentation is the general rule; both of these methods seem well suited to Numbers, and to Naperian logarithms, but are very deficient when applied to common logarithms, on account of their not representing the tail of the logarithm with sufficient exactitude. For example, in a table of five-figure logarithms with marked augmentations we should find

log.	88637	to be	·9476$\underline{2}$
log.	88638	„	·9476$\underline{2}$
log.	88639	„	·94762