

**A SYNOPSIS OF ELEMENTARY RESULTS
IN PURE AND APPLIED MATHEMATICS:
CONTAINING
PROPORTIONS, FORMULAE, AND
METHODS OF ANALYSIS, WITH
ABRIDGED DEMONSTRATIONS. VOL. I**

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A Synopsis of Elementary Results in Pure and Applied Mathematics: Containing Proportions, Formulae, and Methods of Analysis, with Abridged Demonstrations. Vol. I by G. S. Carr

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BY
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VOL. I.



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FLEET STREET.

1880.

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187. *h* 17

1814-17, which give the least divisors of all numbers from 1 to 3,036,000. In a certain sense, it may be said that this is the only sort of purely mathematical table which is absolutely indispensable, because the information which it gives cannot be supplied by any process of direct calculation. The logarithm of a number, for instance, may be computed by a formula. Not so its prime factors. These can only be arrived at through the tentative process of successive divisions by the prime numbers, an operation of a most deterrent kind when the subject of it is a high integer.

A table similar to and in continuation of Burckhardt's has recently been constructed for the fourth million by J. W. L. Glaisher, F.R.S., who I believe is also now engaged in completing the fifth and sixth millions. The factors for the seventh, eighth, and ninth millions were calculated previously by Dase and Rosenberg, and published in 1862-65, and the tenth million is said to exist in manuscript. The history of the formation of these tables is both instructive and interesting.*

As, however, such tables are necessarily expensive to purchase, and not very accessible in any other way to the majority of persons, it seemed to me that a small portion of them would form a useful accompaniment to the present volume. I have, accordingly, introduced the first eleven pages of Burckhardt's tables, which give the least factors of the first 100,000 integers nearly. Each double page of the table here printed is

* See "*Factor Table for the Fourth Million.*" By James Glaisher, F.R.S. London: Taylor and Francis. 1880. Also *Camb. Phil. Soc. Proc.*, Vol. III, Pt. IV., and *Nature*, No. 542, p. 462.

an exact reproduction, in all but the type, of a single quarto page of Burckhardt's great work.

It may be noticed here that Prof. Lebesgue constructed a table to about this extent, on the plan of omitting the multiples of seven, and thus reducing the size of the table by about one-sixth.* But a small calculation is required in using the table which counterbalances the advantage so gained.

The values of the Gamma-Function, pages 30 and 31, have been taken from Legendre's table in his "*Exercices de Calcul Intégral*," Tome I. The table belongs to Part II. of this Volume, but it is placed here for the convenience of having all the numerical tables of Volume I. in the same section.

In addition to the authors already named, the following treatises have been consulted—Algebras, by Wood, Bourdon, and Lefebure de Fourcy; Snowball's Trigonometry; Salmon's Higher Algebra; the Geometrical Exercises in Potts's Euclid; and Geometrical Conics by Taylor, Jackson, and Renshaw.

Articles 260, 431, 569, and very nearly all the examples, are original. The latter have been framed with great care, in order that they might illustrate the propositions as completely as possible.

G. S. C.

HADLEY, MIDDLESEX;
May 23, 1880.

* "*Tables diverses pour la décomposition des nombres en leurs facteurs premiers.*" Par V. A. Lebesgue. Paris. 1864.

ERRATA.

Art. 13,		for	$-a^{2/3}$	read	$+a^{2/3}$.
" 66,	line 1,	"	3	"	$\frac{8}{3}$.
" 66,	" 6,	"	x	"	x^3 .
" 90,	" 4,	"	numerators 1, 1, 1	"	1, a , a^2 .
" 99,	" 1,	"	denominator $r-1$	"	$n-1$.
" 107,	" 1,	"	taken	"	taken as at a time.
" 108,	" 2,	"	(196)	"	(360).
" 131,	" 1, 2,	"	6	"	6.
" "	" 6,	"	$(-1)^6$	"	$(-1)^{10}$.
" 133,	" 3, 6, 7,	"	$6x$	"	$3x$.
" "	" 8,	"	4	"	34.
" "	" 9,	"	204, 459	"	102, 306.
" "	" 10,	"	459	"	9m.
" 138,	" 4,	"	$\frac{10.9.6}{1.2.3}$	"	$7.8.9.10$.
" 140,	"	"	$(q+1)^4$	"	$(q+1)^{(4)}$ Notation of (96).
" 152,	" 6,	"	u_{n-1} in numerator	"	u_{n-1}^2 .
" 191,	" 4,	"	(163)	"	(164).
" 220,	" 6,	"	$(x+y+z)^2$	"	$2(x+y+z)^2$.
" 221,	" 4,	"	(1)	"	square of (1).
" 237,	" 11,	"	$x^2=1$	"	$x^2=-1$.
" 238,	" 6,	"	(x^2-4x+8) on left side	"	$(x^2-4x+8)^2$.
" 239,	" 11,	"	(234)	"	<i>Dele.</i>
" 248,	" 4,	"	(23)	"	(28).
" 267,	" 4,	"	(267)	"	(266).
" 274,	" 8,	"	$\frac{11}{2}$	"	$2\frac{11}{2}$.
" 276,	" 13,	"	$p+2$	"	$p+1$.
" "	" 14,	"	$(p-1)$	"	$\frac{1}{p-1}$.
" 283,	" 3,	"	$x=1$	"	$a=2$.
" 288,	" 7,	"	$n-1$	"	$n+1$.
" 289,	" 4,	"	$H(r, n-1)$	"	$H(n, r-1)$.
" 290,	" 2,	"	$H(r+1, n-1)$	"	$H(n, r)$.
" 325,	" 17,	"	P_2	"	P .
" "	"	"	P_1, P_2, P_3 , last line but one	"	Q_1, Q_2, Q_3 .
" 333,	" 8,	"	$\left(\frac{a+b}{2}\right)$	"	$\left(\frac{a+b}{2}\right)^m$.
" 361,	" 7,	"	3528	"	10284.
" 481,	" 6,	"	$n-3$	"	$n-1$.
" 614,	" 4,	"	applying Descartes' rule	"	<i>Dele.</i>
" 617,	" 3,	"	a^2	"	x^2 .
" 644,	" 1,	"		"	Transpose F and f .
" 661,	" 1,	"	B_1	"	B .
" "	" 9,	"	$a-n$	"	$a-\kappa$.
" 704,	"	"	(11, 12)	"	(9, 10, 1).
" 729,	"	"	(940)	"	(969).

Article 112 should be as follows:—

$$\frac{1}{1+2\sqrt{3}-\sqrt{2}} = \frac{1+2\sqrt{3}+\sqrt{2}}{(1+2\sqrt{3})^2-2} = \frac{1+2\sqrt{3}+\sqrt{2}}{11+4\sqrt{3}} = \frac{(1+2\sqrt{3}+\sqrt{2})(11-4\sqrt{3})}{78}.$$

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