FAMOUS PROBLEMS OF ELEMENTARY GEOMETRY

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WOOSTER WOODRUFF BEMAN & DAVID EUGENE SMITH

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FAMOUS PROBLEMS

OF

ELEMENTARY GEOMETRY

THE DUPLICATION OF THE CUBE
THE TRISECTION OF AN ANGLE
THE QUADRATURE OF THE CIRCLE

AN AUTHORIZED TRANSLATION OF F. KLEIN'S
VORTRÄGE ÜBER AUSGEWÄHLTE FRAGEN DER ELEMENTARGEOMETRIE
AUSGEARBEITET VON F. TÄGERT

BY

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PREFACE.

THE more precise definitions and more rigorous methods of demonstration developed by modern mathematics are looked upon by the mass of gymnasium professors as abstruse and excessively abstract, and accordingly as of importance only for the small circle of specialists. With a view to counteracting this tendency it gave me pleasure to set forth last summer in a brief course of lectures before a larger audience than usual what modern science has to say regarding the possibility of elementary geometric constructions. Some time before, I had had occasion to present a sketch of these lectures in an Easter vacation course at Göttingen. The audience seemed to take great interest in them, and this impression has been confirmed by the experience of the summer semester. I venture therefore to present a short exposition of my lectures to the Association for the Advancement of the Teaching of Mathematics and the Natural Sciences, for the meeting to be held at Göttingen. This exposition has been prepared by Oberlehrer Tägert, of Ems, who attended the vacation course just mentioned. He also had at his disposal the lecture notes written out under my supervision by several of my summer semester students. I hope that this unpretending little book may contribute to promote the useful work of the association.

F. KLEIN.

GÖTTINGEN, Easter, 1895.

TRANSLATORS' PREFACE.

At the Göttingen meeting of the German Association for the Advancement of the Teaching of Mathematics and the Natural Sciences, Professor Felix Klein presented a discussion of the three famous geometric problems of antiquity, — the duplication of the cube, the trisection of an angle, and the quadrature of the circle, as viewed in the light of modern research.

This was done with the avowed purpose of bringing the study of mathematics in the university into closer touch with the work of the gymnasium. That Professor Klein is likely to succeed in this effort is shown by the favorable reception accorded his lectures by the association, the uniform commendation of the educational journals, and the fact that translations into French and Italian have already appeared.

The treatment of the subject is elementary, not even a knowledge of the differential and integral calculus being required. Among the questions answered are such as these: Under what circumstances is a geometric construction possible? By what means can it be effected? What are transcendental numbers? How can we prove that e and π are transcendental?

With the belief that an English presentation of so important a work would appeal to many unable to read the original, Professor Klein's consent to a translation was sought and readily secured.

In its preparation the authors have also made free use of the French translation by Professor J. Griess, of Algiers, following its modifications where it seemed advisable.

They desire further to thank Professor Ziwet for assistance in improving the translation and in reading the proof-sheets.

W. W. BEMAN.
D. E. SMITH.

August, 1897.

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