HINTS AND ANSWERS; BEING A KEY TO A COLLECTION OF CAMBRIDGE MATHEMATICAL EXAMINATION PAPERS, AS PROPOSED AT THE SEVERAL COLLEGES. PART 1. ONTAINING EUCLID, ARTHIMETIC, AND ALGEBRA

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Hints and answers; being a key to a collection of Cambridge mathematical examination papers, as proposed at the several colleges. Part 1. ontaining Euclid, arthimetic, and algebra by J. M. F. Wright

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J. M. F. WRIGHT

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Trieste

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TO A COLLECTION OF

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AS PROPOSED AT

THE SEVERAL COLLEGES.

BY

J. M. F. WRIGHT, B. A.

AUTHOR OF THE PRIVATE TUTOR, A TRANSLATION OF NEWTON'S PRINCIPIA,
SECTIONS I. II. III., &c. &c. &c.

PART I.

CONTAINING EUCLID, ARITHMETIC, AND ALGEBRA.

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1831.

PREFACE.

This work comprises not only *Hints* for the solution of the more difficult parts of questions, but *Answers* for all, over and above the complete developement of many of the most intricate. The full proof, or investigation, is bestowed upon nearly all the last five Algebraic Problems and Equations in each of those papers given at St. John's College which invariably consists of seven questions,—these papers having been found far more difficult of treatment than any proposed at other Colleges. Interesting questions also have been somewhat fully solved and exemplified; and the shortest practical methods have constantly been suggested on every available occasion. Notwithstanding so much has been done for the student, the work does not greatly exceed double the bulk of the questions themselves.

It were almost superfluous to direct students in the proper use of these aids. Let no Hint, Answer, or Solution be referred to, before the student's own patient efforts have entirely failed. A reference then may be equally advantageous with the help of a tutor. But less benefit will accrue in proportion to the contrary being practised. To boys at school, whose only aim is to get through their tasks, Keys are detrimental; but to students at an University, where high honours and rewards await distinguished success at Examinations, the motives being essentially different, they will be rendered to good account—at least, by those who value their own progress in knowledge.

As the subjects of Geometry or Euclid, Arithmetic and Algebra, form a principal part of the studies for the degree of B.A., as well as for the Examination of Freshmen, it is presumed the volume will not prove unwelcome to those Undergraduates, who are considered the "*non-reading*" portion of the University.

Another volume, containing "Hints and Answers in *Trigonometry, and the Differential Calculus,*" is in progress, and will speedily be published.

Trinity House, Christ's Piece, Cambridge,
March 1, 1831.

BOOKS REFERRED TO.

Euclid's Elements. *Simson's.*
Wright's Self-Examinations in Euclid.
Creswell's Maxima and Minima.
Bland's Geometrical Problems.
Wright's Pure Arithmetic.
Wood's Algebra.
Wright's Private Tutor.
Barlow's Theory of Numbers.

HINTS AND ANSWERS

IN

EUCLID, ARITHMETIC, & ALGEBRA.

EUCLID.

TRINITY COLLEGE, 1824.

[P. 1.

1. The principles of construction are the three Postulates of Euclid.

2. Euclid, Book i. prop. 4.

3. Take any right-angled △ ABC, right-angled at C; from C draw CM ⊥ AB, thus obtaining three right-angled △s. Then, since the △s ACM, ACB, have two angles in each equal, their third angles are equal; that is, ∠ ACM = ∠ B. Similarly, it may be shown that ∠ BCM = ∠ A.

Hence, ∠ A + ∠ B = ∠ ACM + ∠ BCM = a right angle. *Consequently the two angles of a right-angled △ which are not the right angle*, are equal to one right angle.

Again, take any △ whatever abc, and from c draw $cm \perp ab$; then, by what has been shown,

$$\angle a + \angle acm = \text{one right } \angle,$$
$$\text{and } \angle b + \angle bcm = \text{one right } \angle,$$
$$\therefore \angle a + \angle b + \angle c = \text{two right } \angle\text{s}.$$

4. Euclid, i. 35.

5. Let P be the given point in the side AB of the given △ ABC. Suppose the thing done, and that the required line, bisecting the △, meets the side BC in the point Q; then the △ BPQ = $\frac{1}{2}$ △ ABC.

Complete the parallelogram BPQR. This parallelogram $= 2 \triangle$ BPQ $= \triangle$ ABC. Whence the Synthetic construction and demonstration are evident.

For upon BP describe any parallelogram equal to the $\triangle$ ABC, (Euclid, i. 44), and let Q be the point where the upper side of the parallelogram enters the side BC. Join PQ. Then $\because$ &c. as is evident.

6. Euclid, i. 46, and *Wright's* Self-Exam. in Euclid, p. 4.

7. Euclid, i. 47. 8. Euclid, ii. 14. 9. Euclid, iii. 2.

10. *Wood*, Alg. art. 515. 11. Euclid, iv. 11.

12. They are an equilateral $\triangle$, a square, and a regular hexagon.

13. For problems of this kind, see *Creswell's* Maxima and Minima.

14. See *Wright's* Self-Exam. in Euclid, pp. 73, 74, 75, 76.

15. Euc. v. 12. 16. Euc. vi. 1. 17. Euc. vi. 4. 18. Euc. vi. 25.

19. To the side C of the rectangle $A \times C$ apply a rectangle $= B \times D$; then the base of $A \times C$ has to the base of the whole rectangle the ratio required.

20. For if the straight lines AB, CD cut the given lines AC, BD in A, B; C, D, respectively, and intersect in P; and likewise two other straight lines A'B', C'D', *parallel* to the former, cut the same given lines in A', B', and C', D', respectively, and intersect in P'; then the $\triangle$s APC, A'P'C' are similar, and also DPB, and D'P'B'. $\therefore$ &c.

21. There are not data sufficient that the square shall be constructed *in position*, but only in magnitude; for it is clear the square, when constructed, from these given distances, might all assume any position around the point.

If a, b, be the two shorter distances, and c the longest; then it may easily be shown that the side of the square is

$$\sqrt{\frac{b^2+c^2 \pm \sqrt{\{4b^2c^2-(2a^2-b^2-c^2)^2\}}}{2}},$$

whence the construction is easy.

One solution belongs to that case in which the given point lies without the square; the other to that in which it lies within the square.

The problem is impossible when

$$2a^2-b^2-c^2 \text{ is } > 2bc,$$
$$\text{or when } a \text{ is } > \frac{b+c}{\sqrt{2}},$$

or when $\frac{a}{b+c}$ is $> \frac{1}{\sqrt{2}} >$ ratio of the side of a square to its diagonal.

This is also evident, from the consideration that two sides of a △ must be always greater than the third side.

22. Join the two given points A, B; bisect AB by the perpendicular ED, meeting the given line CD in D. Then the centre of the required circle is in ED; and the distance of its centre O from D is

$$\frac{DE^3 \pm DE\sqrt{(DE^2 . EF^2 + EF^2 . EB^2 - ED^2 . EF^2)}}{DE^2 - EF^2},$$

EF making an ∠ with CD equal the given ∠.

23. Let AB be any side of the regular polygon, C being the centre of the circumscribed circle. Bisect the ∠ A′CB by the straight line CM; then AM = MB. Again, bisect the ∠ MCB by Cn, and draw Cm', Cb' ⊥ Cn, and make each of them $= \frac{1}{2}$ MB. Draw $m'm$, $b'b$ parallel to Cn, to meet CM, CB in m, b; join mb; then, it is easily shown, that $mb =$ MB. In like manner draw $ma =$ AM; am, mb, will be two sides of the new regular polygon; and, similarly, all the others may be found.

TRINITY COLLEGE, 1828.

1. Euc. i. 24. 2. Euc. i. 45. 3. Euc. ii. 7. 4. Euc. ii. 11. 5. Euc. iii. 27. 6. Euc. iii. 33. 7. Euc. iv. 5. 8. Euc. iv. 13. 9. Euc. v. 10. 10. Euc. v. 22. 11. Euc. vi. 22. 12. Euc. vi. 33.

13. Let the straight lines AC, BD, cut by AB, make the angles BAC, ABD together less than two right angles. Draw

AE, making the angle BAE + angle ABD equal two right angles; then, by the assumption, AE and BD cannot meet. Consequently, AC and BD will meet.

14. This is easily proved, in a manner similar to that of Prop. 47. B. i. of Euclid.

15. First, make a square = given rectangle (Euclid, ii. 14). Next, on the given line describe a semi-circle; at the extremity of the line draw a ⊥ which is = the side of the square; from the end of this ⊥ draw a line ⊥ to it meeting the semicircle in a point from which the ⊥ upon the given line will divide it as required.

16. See *Bland's* Geomet. Problems, p. 231; or make use of Euclid, iv. 10.

17. This depends upon Euclid, ii. 12 and 13, and upon B. vi. prop. B.

18. Let the chords AB, ab, be produced to C; and let O be the centre of the circle; join OA, OB, Oa, Ob, OC, and produce CO to the circumference in D.

$$\text{Then } \angle DOA = \angle OAC + \angle OCA = \angle OBA + \angle OCA,$$
$$= \angle BOC + 2\angle OCA,$$
$$\therefore \frac{\angle DOA - \angle BOC}{2} = \angle OCA;$$

&c.

19. From one extremity A of the base of AB of the △ as a centre and a radius = sum of the undetermined sides describe an arc. From the other extremity B draw any line BC to the circumference; join AC, and make ∠ CBD = BCA, &c.

See also *Creswell's* Maxima and Minima.

20. Let the straight line a be the side of the given square; straight lines b, c, denote the antecedent and consequent of the given ratio; take a fourth proportional to c, a, b, which is $a.\dfrac{b}{c}$; and find a mean proportional to a, and $a.\dfrac{b}{c}$; this mean proportional will be the side of the square required.