ON THE STABILITY OF THE MOTION OF SATURN'S RINGS

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649368082

On the stability of the motion of Saturn's rings by J. Clerk Maxwell

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

www.triestepublishing.com

J. CLERK MAXWELL

ON THE STABILITY OF THE MOTION OF SATURN'S RINGS

Trieste

ON THE

STABILITY OF THE MOTION

OF

SATURN'S RINGS.

AN ESSAY,

WHICH OBTAINED THE ADAMS PRIZE FOR THE YEAR 1856, IN THE UNIVERSITY OF CAMBRIDGE.

BY J. CLERK MAXWELL, M.A.

LATE FELLOW OF TRINITY COLLEGE, CAMBRIDGE.
PROFESSOR OF NATURAL PHILOSOPHY
IN THE MARISCHAL COLLEGE AND UNIVERSITY OF ABERDEEN.

"E pur si muove."

Cambridge:
MACMILLAN AND CO.
AND 23 HENRIETTA STREET, COVENT GARDEN, LONDON.
1859.

Cambridge:
PRINTED BY C. J. CLAY, M.A.
AT THE UNIVERSITY PRESS.

ADVERTISEMENT.

THE Subject of the Prize was announced in the following terms:—

> The University having accepted a fund raised by several members of St John's College, for the purpose of founding a Prize to be called the ADAMS PRIZE, for the best Essay on some subject of Pure Mathematics, Astronomy, or other branch of Natural Philosophy, the Prize to be given once in two years, and to be *open to the competition of all persons who have at any time been admitted to a degree in this University*:—

The Examiners give Notice, that the following is the subject for the Prize to be adjudged in 1857:—

The Motions of Saturn's Rings.

*** The Problem may be treated on the supposition that the system of Rings is exactly or very approximately concentric with Saturn and symmetrically disposed about the plane of his Equator, and different hypotheses may be made respecting the physical constitution of the Rings. It may be supposed (1) that they are rigid: (2) that they are fluid, or in part seriform: (3) that they consist of masses of matter not mutually coherent. The question will be considered to be answered by ascertaining on these hypotheses severally, whether the conditions of mechanical stability are satisfied by the mutual attractions and motions of the Planet and the Rings.

It is desirable that an attempt should also be made to determine on which of the above hypotheses the appearances both of the bright Rings and the recently discovered dark Ring may be most satisfactorily explained; and to indicate any causes to which a change of form, such as is supposed from a comparison of modern with the earlier observations to have taken place, may be attributed.

E. GUEST, *Vice-Chancellor.*
J. CHALLIS.
S. PARKINSON.
W. THOMSON.

March 23, 1855.

CONTENTS.

PAGE

Nature of the Problem 1

Laplace's investigation of the Equilibrium of a Ring, and its minimum density . . 2

His proof that the plane of the Rings will follow that of Saturn's Equator—that a solid uniform Ring is necessarily unstable 3

Further investigation required—Theory of an Irregular Solid Ring leads to the result that to ensure stability the irregularity must be enormous 4

Theory of a fluid or discontinuous Ring resolves itself into the investigation of a series of waves 4

PART I.

ON THE MOTION OF A RIGID BODY OF ANY FORM ABOUT A SPHERE.

Equations of Motion 6

PROBLEM I. *To find the conditions under which a uniform motion of the Ring is possible* 8

PROBLEM II. *To find the equations of the motion when slightly disturbed* . . . 8

PROBLEM III. *To reduce the three simultaneous equations of motion to the form of a single linear equation* 9

PROBLEM IV. *To determine whether the motion of the Ring is stable or unstable, by means of the relations of the coefficients* A, B, C 10

PROBLEM V. *To find the centre of gravity, the radius of gyration, and the variations of the potential near the centre of a circular ring of small but variable section* . 11

PROBLEM VI. *To determine the condition of stability of the motion in terms of the coefficients* f, g, h, *which indicate the distribution of mass in the ring* . . . 14

RESULTS. *1st, a uniform ring is unstable. 2nd, a ring varying in section according to the law of sines is unstable. 3rd, a uniform ring loaded with a heavy particle may be stable, provided the mass of the particle be between* ·315885 *and* ·3279 *of the whole. Case in which the ring is to the particle as* 18 *to* 82 15

PART II.

ON THE MOTION OF A RING, THE PARTS OF WHICH ARE NOT RIGIDLY CONNECTED.

1. *General Statement of the Problem, and limitation to a nearly uniform ring* . 18
2. *Notation* 18
3. *Expansion of a function in terms of sines and cosines of multiples of the variable* 19
4. *Magnitude and direction of attraction between two elements of a disturbed Ring* . 19
5. *Resultant attractions on any one of a ring of equal satellites disturbed in any way* 20

Note. *Calculated values of these attractions in particular cases* . . . 21

PAGE

6. *Equations of motion of a satellite of the Ring, and biquadratic equation to determine the wave-velocity* 21
7. *A ring of satellites may always be rendered stable by increasing the mass of the central body* 23
8. *Relation between the number and mass of satellites and the mass of the central body necessary to ensure stability.* $S > \cdot 4852\ \mu^2 R$ 24
9. *Solution of the biquadratic equation when the mass of the ring is small; and complete expressions for the motion of each satellite* 52
10. *Each satellite moves (relatively to the ring) in an ellipse* 26
11. *Each satellite moves absolutely in space in a curve which is nearly an ellipse for the large values of* n, *and a spiral of many nearly circular coils when* n *is small* 27
12. *The form of the ring at a given instant is a series of undulations* . . . 27
13. *These undulations travel round the ring with velocity* $-\frac{n}{m}$ *relative to the ring, and* $\omega - \frac{n}{m}$ *absolutely* 28
14. *General Solution of the Problem—Given the position and motion of every satellite at any one time, to calculate the position and motion of any satellite at any other time, provided that the condition of stability is fulfilled* 28
15. *Calculation of the effect of a periodic external disturbing force* 30
16. *Treatment of disturbing forces in general* 32
17. *Theory of free waves and forced waves* 33
18. *Motion of the ring when the conditions of stability are not fulfilled. Two different ways in which the ring may be broken up* 34
19. *Motion of a ring of unequal satellites* 37
20. *Motion of a ring composed of a cloud of scattered particles* 38
21. *Calculation of the forces arising from the displacements of such a system* . . 39
22. *Application to the case of a ring of this kind. The mean density must be excessively small, which is inconsistent with its moving as a whole* 39
23. *On the forces arising from inequalities in a thin stratum of gravitating incompressible fluid of indefinite extent* 40
24. *Application to the case of a flattened fluid ring, moving with uniform angular velocity. Such a ring will be broken up into portions which may continue to revolve as a ring of satellites* 45

ON THE MUTUAL PERTURBATIONS OF TWO RINGS.

25. *Application of the general theory of free and forced waves* 45
26. *To determine the attractions between the rings* 46
27 *To form the equations of motion* 48
28. *Method of determining the reaction of the forced wave on the free wave which produced it* 49
29. *Cases in which the perturbations increase indefinitely* 50
30. *Application to the theory of an indefinite number of concentric rings* . . . 51

PAGE

31. *On the effect of long continued disturbances on a system of Rings* . . . 51
32. *On the effect of collisions among the parts of a revolving system* 52
33. *On the effect of Internal Friction in a Fluid Ring* 53
Recapitulation of the Theory of the Motion of a Rigid Ring, Reasons for rejecting the hypothesis of rigidity 55
Recapitulation of the Theory of a Ring of Equal Satellites 57
Description of a working model shewing the motions of such a system 59
Theory of rings of various constitutions 64
Mutual action of Two Rings 65
Case of many concentric Rings, &c 66
General Conclusions 66
APPENDIX. *Extract of a letter from Professor W. Thomson, of Glasgow, giving a solution of the Problem of a Rigid Ring* 69