

**AN ELEMENTARY TREATISE ON  
THE LUNAR THEORY, WITH A  
BRIEF SKETCH OF THE HISTORY  
OF THE PROBLEM BEFORE  
NEWTON**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649542079

An Elementary Treatise on the Lunar Theory, with a Brief Sketch of the History of the Problem before Newton by Hugh Godfray

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.  
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

[www.triestepublishing.com](http://www.triestepublishing.com)

**HUGH GODFRAY**

**AN ELEMENTARY TREATISE ON  
THE LUNAR THEORY, WITH A  
BRIEF SKETCH OF THE HISTORY  
OF THE PROBLEM  
BEFORE NEWTON**



**AN ELEMENTARY  
TREATISE ON THE LUNAR THEORY.**

AN  
ELEMENTARY TREATISE  
ON THE  
LUNAR THEORY,

WITH  
A BRIEF SKETCH OF THE HISTORY OF THE PROBLEM  
BEFORE NEWTON.

BY  
HUGH GODFRAY, M.A.  
OF ST. JOHN'S COLLEGE, CAMBRIDGE;  
LATE MATHEMATICAL LECTURER AT PEMBROKE COLLEGE.

FOURTH EDITION.



London and New York:  
MACMILLAN AND CO.

1885.

ck

QB 391  
66

## PREFACE TO THE FIRST EDITION.

OF all the celestial bodies whose motions have formed the subject of the investigations of astronomers, the Moon has always been regarded as that which presents the greatest difficulties, on account of the number of inequalities to which it is subject; but the frequent and important applications of the results render the Lunar Problem one of the highest interest, and we find that it has occupied the attention of the most celebrated astronomers from the earliest times.

Newton's discovery of Universal Gravitation, suggested, it is supposed, by a rough consideration of the motions of the Moon, led him naturally to examine its application to a more severe explanation of her disturbances; and his Eleventh Section is the first attempt at a theoretical investigation of the Lunar inequalities. The results he obtained were found to agree very nearly with those determined by observation, and afforded a remarkable confirmation of the truth of his great principle; but the geometrical methods which he had adopted seem inadequate to so complicated a theory, and recourse has been had to analysis for a complete determination of the disturbances, and for a knowledge of the true orbit.

The following pages will, it is hoped, form a proper introduction to more recondite works on the subject: the difficulties which a person entering upon this study is most likely to stumble at have been dwelt upon at considerable length, and though different methods of investigation have been employed by different astronomers, the difficulties met with are nearly the same, and the principle of successive approximation is common to all. In the present work, the approximation is carried to the second order of small quantities, and this, though far from giving accurate values, is amply sufficient for the elucidation of the method.

The differences in the analytical solutions arise from the various ways in which the position of the moon may be indicated by altering the system of coordinates to which it is referred, or again, in the same system, by choosing different quantities for independent variables.

D'Alembert and Clairaut chose for coordinates the projection of the radius vector on the plane of the ecliptic and the longitude of this projection. To form the differential equations, the true longitude was taken for independent variable.

To determine the latitude, they, by analogy to Newton's method, employed the differential variations of the motion of the node and of the inclination of the orbit.

Laplace, Damoiseau, Plana, and also Herschel and Airy, in their more elementary works, have found it more convenient to express the variations of the latitude directly, by an equation of the same form as that of the radius vector.

Lubbock and Pontécoulant, taking the same coordinates of the Moon's position, make the time the independent



variable; and when it is desired to carry the approximation to a high order, this method offers the advantage of not requiring any reversion of series.

Poisson proposed the method used in the planetary theory, that is, to determine the variation in the elements of the Moon's orbit, and thence to conclude the corresponding variations of the radius vector, the longitude, and the latitude.

The selection of the method followed in the present-work, which is the same as that of Airy, Herschel, &c., was made on account of its simplicity; moreover, it is the method which has obtained sanction in this University, and it is hoped that it may prove of service to the student in his reading for the examination for Honours. In furtherance of this object, one of the chapters (the sixth) contains the physical interpretation of the various important terms in the radius vector, latitude, and longitude.\*

The seventh chapter, or appendix, contains some of the most interesting results in the terms of the higher orders, among which will be found the values of  $c$  and  $g$  completely obtained to the third order.

The last chapter is a brief historical sketch of the Lunar Problem before Newton, containing an account of the discoveries of the several inequalities and of the methods by which they were represented, those only being mentioned which, as the theory has since verified, were real onward steps. The perusal of this chapter will shew to what extent we are indebted to our great philosopher; at the same time we cannot fail being impressed with reverence for the

---

\* See the Report of the "Board of Mathematical Studies" for 1850.

genius and preservance of the men who preceded him, and whose elaborate and multiplied hypotheses were in some measure necessary to the discovery of his simple and single law.

I take this opportunity of acknowledging my obligations to several friends, whose valuable suggestions have added to the utility of the work.

HUGH GODFRAY.

*St. John's College, Cambridge,*  
*April, 1868.*

---

#### PREFACE TO THE THIRD EDITION.

In the present edition several important alterations have been made, many of the parts have been entirely re-written, and a general re-arrangement of the whole subject has taken place. In Chap. iv. Sect. 3, the necessity for the computation of the elements in the order  $s, u, \theta$  is shewn. In the *Quarterly Journal*, Mr. Walton pointed out the logical necessity for having the value of  $u$  completely correct to the second order before proceeding to find  $\theta$ , although the result is the same as when the preparation for both values is carried on simultaneously. I have here proved that there is the same necessity for making the value of  $s$  precede that of  $u$ , although, again, the change has no effect on the result.

The term of the third order "Parallactic Inequality" will be found completely investigated in the appendix; and at the end of the volume a selection of questions from University and College Examination Papers has been introduced.

*August, 1871.*

# CONTENTS.

---

## CHAPTER I.

### INTRODUCTION.

Arts.		Page
1	Newton's Law of Universal Attraction . . . . .	1
2-5	Principle of Superposition of Small Disturbances . . . . .	4
6-8	Attractions of Spherical Bodies . . . . .	6

## CHAPTER II.

### MOTION RELATIVE TO THE EARTH.

9-13	Problem of Two Bodies . . . . .	9
14-18	Problem of Three Bodies . . . . .	13
15	Path of Centre of Gravity of Earth and Moon, Ecliptic . . . . .	14
17	Errors in the Sun's assumed place . . . . .	16

## CHAPTER III.

### DIFFERENTIAL EQUATIONS.

19, 20	Formation of the Differential Equations . . . . .	19
21, 22	Orders of the Small Quantities introduced . . . . .	21
23, 24	Values of the Forces $P, T, S$ . . . . .	23
25	Order of the Sun's Disturbing Force . . . . .	26

## CHAPTER IV.

### INTEGRATION OF THE DIFFERENTIAL EQUATIONS.

26	General Process described . . . . .	27
27-30	Terms of a higher order which must be retained . . . . .	28
32-36	Solution to the first order . . . . .	32
37-42	Introduction of $c$ and $g$ . . . . .	34