THE ELEMENTS OF SPHERICAL TRIGONOMETRY

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649344079

The Elements of Spherical Trigonometry by James Hann

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

www.triestepublishing.com

JAMES HANN

THE ELEMENTS OF SPHERICAL TRIGONOMETRY

Trieste

THE ELEMENTS

OF

SPHERICAL TRIGONOMETRY.

BY JAMES HANN,

MATHEMATICAL MASTER OF KING'S COLLEGE SCHOOL, LONDON;

HONORARY MEMBER OF THE PHILOSOPHICAL SOCIETY OF NEWCASTLE-UPON-TYNE,
JOINT AUTHOR OF MECHANICS FOR PRACTICAL MEN,
AUTHOR OF THEORY OF BRIDGES, JOINT AUTHOR OF NAUTICAL TABLES,
AUTHOR OF A TREATISE ON THE STEAM ENGINE,
AUTHOR OF THEORETICAL AND PRACTICAL MECHANICS,
AND AUTHOR OF PLANE TRIGONOMETRY.

LONDON:
JOHN WEALE, 59, HIGH HOLBORN.

1849.

ERRATA TO THE *PLANE* TRIGONOMETRY.

Page 10, Line 5, *for* AP *read* AP^2.
" 9, *for* DP *read* DP^2.
53, " 3 from top, *for* sin 2 A *read* cos 2 A.
10, *for* $\cos\frac{A}{2}$ *read* $\sqrt{2}.\cos\frac{A}{2}$.
" 11 *for* $\text{cosec}^2 A$ *read* $\text{cosec}^2 2A$.
" 12 *for* $-\sqrt{-1}$ *read* $-\sqrt{-1}.\sin A$.
77, Line 24, *for* $\frac{\sqrt{3}}{2}$ *read* $\frac{\sqrt{3}}{4}$.
93, " 2, *for* $24r\frac{180}{24}$ *read* $24r\sin\frac{180}{24}$.
100, " 3, *for* $ac(\theta+\phi)$ *read* $ac\sin(\theta+\phi)$.
102, Fig. 2, *for* C *read* D; *and for* D *read* C.
103, Line 5 from bottom, *for* $c\cos\beta$ *read* $c\cos\gamma$.

PREFACE.

In the compilation of this work, the most esteemed writers, both English and foreign, have been consulted, but those most used are De Fourcy and Legendre.

Napier's Circular Parts have been treated in a manner somewhat different to most modern writers. The terms conjunct and adjunct, used by Kelly and others, are here retained, as they appear to be more conformable to the practical views of Napier himself.

There are many other parts connected with Spherics that might be treated of, but which are not adapted to a Rudimentary Treatise like the present; those, however, who wish to see all the higher departments fully developed, must consult the writings of that distinguished mathematician, Professor Davies, of the Royal Military Academy, Woolwich.

Hutton's Course, the Ladies' and Gentleman's Diaries, (latterly comprised in one), Leybourne's Repository, the Mechanics' Magazine, and various other periodicals, teem with the productions of his fertile mind, both on this and other kindred subjects.

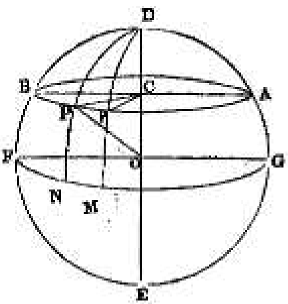
CONTENTS.

SPHERICS.

PRELIMINARY CHAPTER.

1. A SPHERE is a solid determined by a surface of which all the points are equally distant from an interior point, which is called the centre of the sphere.

2. Every section of a sphere made by a plane cutting it is the arc of a circle.



Let O be the centre of the sphere, APBA a section made by a plane passing through it, draw OC to the cutting plane, and produce it both ways to D and E, and draw the radii of the sphere OA, OP.

Now, since OCP and OCA are right angles, $OA^2 - OC^2 = AC^2$, and $OP^2 - OC^2 = PC^2$, but $OA^2 = OP^2$; $\therefore AC^2 = PC^2$ or $AC = PC$; hence the section APBA is a circle.

If the cutting plane pass through the centre, the radius of the section is evidently equal to the radius of the sphere, and such a section is called a great circle of the sphere.

3. The poles of any circle are the two extremities of that diameter or axis of the sphere which is perpendicular to the plane of that circle; and therefore either pole of any circle is equidistant from every part of its circumference, and, if it be a great circle, its pole is 90° from the circumference. A spherical triangle is the portion of space comprised between three arcs of intersecting great circles.

4. The angles of a spherical triangle are those on the surface of the sphere contained by the arcs of the great circles which form the sides, and are the same as the inclinations of the planes of those great circles to one another.

5. Any two sides of a spherical triangle are greater than the third side.

Since by Euclid XI. 20, any two of the plane angles, which form the solid angle at O, are together greater than the third, hence any two of the arcs which measure those angles must be greater than the third.

6. Since the solid angle at O (see fig. p. 3) is contained by three plane angles, and by Euclid XI. 21, these are together less than four right angles, hence the three arcs of the spherical triangle which measure those angles must be together less than the circumference of a great circle, that is $a + b + c > 360$, and since any two sides of a triangle is greater than the third, we have $a + b > c$; $b + c > a$; $a + c > b$.

ON THE POLAR OR SUPPLEMENTAL TRIANGLE.

7. If three arcs of great circles be described from the angular points A, B, C, of any spherical triangle A B C, as poles, the sides and angles of the new triangle, D F E, so formed will be the supplements of the opposite angles and sides of the other, and *vice versâ*.

Since B is the pole of D F, then B D is a quadrant, and since C is the pole of D E, C D is a quadrant; therefore the distances of the points B and C from D being each a quadrant, they are equal to each other, hence D is the pole of B C.

$DE = 180° - C$; $EF = 180° - A$;

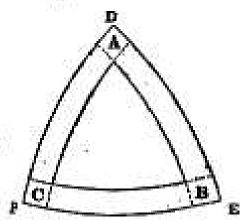
$FD = 180° - B$; and $D = 180° - BC$;

$E = 180° - AC$; $F = 180° - AB$.

Also, $AB = 180° - F$; $BC = 180° - D$;

$AC = 180° - E$; $A = 180 - FE$;

$B = 180° - FD$; $C = 180° - DE$.



The sum of the three angles of a spherical triangle is greater than two right angles, and less than six right angles.

For if $a' + b' + c'$ be the sides of the supplemental or polar triangle, $A = 180° - a'$; $B = 180° - b'$; $C = 180° - c'$;

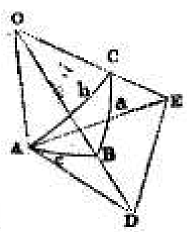
$$\text{hence } A + B + C + a' + b' + c' = 6 \times 90 = 6 \text{ right angles;}$$

but $a' + b' + c'$ is less than four right angles, by Euclid XI. 21; therefore $A + B + C$ is greater than two right angles; and as the sides a', b', c', of the polar triangle must have some magnitude, the sum of the three angles A, B, C must be less than six right angles.

SPHERICAL TRIGONOMETRY.

CHAPTER I.

8. Spherical Trigonometry treats of the various relations between the sines, tangents, &c., of the known parts of a spherical triangle, and those that are unknown; or, which is the same thing, it gives the relations between the parts of a solid angle formed by the inclination of three planes which meet in a point, for the solid angle is composed of six parts, the inclinations of the three plane faces to each other, and also the inclinations of the three edges; in fact, a work might be written on this subject without using the spherical triangle at all, for the six parts of the spherical triangle are measures of the six parts of the solid angle at O. See fig.



9. If a spherical triangle have one of its angles a right angle, it is called a right-angled triangle; if one of its sides be a quadrant, it is called a quadrantal triangle; if two of the sides be equal, it is called an isosceles triangle, &c., as in Plane Trigonometry.

10. To determine the sines and cosines of a spherical triangle in terms of the sines and cosines of the sides.

Let O be the centre of the sphere on which the triangle ABC is situated, draw the radii OA, OB, OC; from OA draw the perpendiculars AD and AE, the one in the plane OAB, and the other in the plane OAC, and suppose them to