AN EXPLANATION OF THE OBSERVED IRREGULARITIES IN THE MOTION OF URANUS

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AN EXPLANATION

OF THE

OBSERVED IRREGULARITIES

IN THE

MOTION OF URANUS,

ON THE HYPOTHESIS OF DISTURBANCES

CAUSED BY A MORE DISTANT PLANET;

WITH A DETERMINATION OF THE MASS, ORBIT, AND POSITION OF THE DISTURBING BODY.

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(*From the Appendix to the Nautical Almanac for the Year* 1851.)

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ADVERTISEMENT.

This Paper was communicated by the Author to the Royal Astronomical Society, and was read to that body, at their ordinary meeting, on November 13, 1846. The Press of the Society being engaged on an extensive paper, on the longitude of Valentia, by the Astronomer Royal, and it being deemed of national importance that Mr. Adams's Paper should be submitted to the world without loss of time, application was made to Capt. W. H. Smyth, R.N., President, and to the Rev. R. Sheepshanks, Secretary, of the Society, who, with their usual promptitude and zeal, granted permission for the immediate printing and publishing of the Paper by the Nautical Almanac Office; and it is under these circumstances that the investigations of Mr. Adams first appear as an extract from the Appendix to the Nautical Almanac for 1851.

W. S. STRATFORD,
Superintendent of the Nautical Almanac.

Nautical Almanac Office,
3, Verulam Buildings, Gray's Inn, London.
December 31, 1846.

ON THE PERTURBATIONS OF URANUS.

1. The irregularities in the motions of Uranus have for a long time engaged the attention of Astronomers. When the path of the planet became approximately known, it was found that, previously to its discovery by Sir W. Herschel in 1781, it had several times been observed as a fixed star by Flamsteed, Bradley, Mayer, and Lemonnier. Although these observations are doubtless very far inferior in accuracy to the modern ones, they must be considered valuable, in consequence of the great extension which they give to the observed arc of the planet's orbit. Bouvard, however, to whom we owe the Tables of Uranus at present in use, found that it was impossible to satisfy these observations, without attributing much larger errors to the modern observations than they admit of, and consequently founded his Tables exclusively on the latter. But in a very few years sensible errors began again to show themselves, and though the Tables were formed so recently as 1821, their error at the present time exceeds two minutes of space, and is still rapidly increasing. There appeared, therefore, no longer any sufficient reason for rejecting the ancient observations, especially since, with the exception of Flamsteed's first observation, which is more than twenty years anterior to any of the others, they are mutually confirmatory of each other.

2. Now that the discovery of another planet has confirmed in the most brilliant manner the conclusions of analysis, and enabled us with certainty to refer these irregularities to their true cause, it is unnecessary for me to enter at length upon the reasons which led me to reject the various other hypotheses which had been formed to account for them. It is sufficient to say, that they all appeared to be very improbable in themselves, and incapable of being tested by any exact calculation. Some had even supposed that at the great distance of Uranus from the Sun, the law of attraction becomes different from that of the inverse square of the distance. But the law of gravitation was too firmly established for this to be admitted, till every other hypothesis had failed, and I felt convinced that in this, as in every previous instance of the kind, the discrepancies which had for a time thrown doubts on the truth of the law, would eventually afford the most striking confirmation of it.

3. My attention was first directed to this subject several years since, by reading Mr. Airy's valuable Report on the recent progress of Astronomy. I find among my

papers the following memorandum, dated July 3, 1841:—"Formed a design, in the beginning of this week, of investigating, as soon as possible after taking my degree, the irregularities in the motion of Uranus which are yet unaccounted for; in order to find whether they may be attributed to the action of an undiscovered planet beyond it, and if possible, thence to determine approximately the elements of its orbit, &c., which would probably lead to its discovery." Accordingly, in 1843, I attempted a first solution of the problem, assuming the orbit to be a circle, with a radius equal to twice the mean distance of Uranus from the Sun. Some assumption as to the mean distance was clearly necessary in the first instance, and Bode's law appeared to render it probable that the above would not be far from the truth. This investigation was founded exclusively on the modern observations, and the errors of the Tables were taken from those given in the Equations of Condition of Bouvard's Tables as far as the year 1821, and subsequently from the observations given in the *Astronomische Nachrichten*, and from the Cambridge and Greenwich Observations. The result showed that a good general agreement between theory and observation might be obtained; but the larger differences occurring in years where the observations used were deficient in number, and the Greenwich Planetary Observations being then in process of reduction, I applied to Mr. Airy, through the kind intervention of Professor Challis, for the observations of some years in which the agreement appeared least satisfactory. The Astronomer Royal, in the kindest possible manner, sent me, in February 1844, the results of all the Greenwich Observations of Uranus.

4. Meanwhile the Royal Academy of Sciences of Göttingen had proposed the Theory of Uranus as the subject of their mathematical prize, and although the little time which I could spare from important duties in my college prevented me from attempting the complete examination of the theory, which a competition for the prize would have required, yet this fact, together with the possession of such a valuable series of observations, induced me to undertake a new solution of the problem. I now took into account the most important terms depending on the first power of the eccentricity of the disturbing planet, retaining the same assumption as before with respect to the mean distance. For the modern observations, the errors of the Tables were taken exclusively from the Greenwich Observations as far as the year 1830, with the exception of an observation by Bessel, in 1823; and subsequently from the Cambridge and Greenwich Observations, and those given in various numbers of the *Astronomische Nachrichten*. The errors of the Tables for the ancient Observations were taken from those given in the Equations of Condition of Bouvard's Tables. After obtaining several solutions differing little from each other, by gradually taking into account more and more terms of the series expressing the Perturbations, I communicated to Professor Challis, in September 1845, the final values which I had obtained for the mass, heliocentric longitude, and elements of the orbit of the assumed planet. The same results, slightly corrected, I communicated in the following month to the Astronomer Royal. The eccentricity coming out much larger than was probable, and later observations showing that the theory founded on the first hypothesis as to the mean distance, was still sensibly in error, I afterwards repeated my investigation, supposing the mean distance to be about $\frac{1}{30}$th part less than before. The result,

which I communicated to Mr. Airy, in the beginning of September of the present year, appeared more satisfactory than my former one, the eccentricity being smaller, and the errors of theory, compared with late observations, being less, and led me to infer that the distance should be still further diminished.

5. In November, 1845, M. Le Verrier presented to the Royal Academy of Sciences at Paris, a very complete and elaborate investigation of the Theory of Uranus, as disturbed by the action of Jupiter and Saturn, in which he pointed out several small inequalities which had previously been neglected; and in June, of the present year, he followed up this investigation by a memoir, in which he attributed the residual disturbances to the action of another planet at a distance from the Sun equal to twice that of Uranus, and found a longitude for the new planet agreeing very nearly with the result which I had obtained on the same hypothesis. On the 31st of August he presented to the Academy a more complete investigation, in which he determined the mass and the elements of the orbit of the new planet, and also obtained limiting values of the mean distance and heliocentric longitude. I mention these dates merely to show that my results were arrived at independently, and previously to the publication of those of M. Le Verrier, and not with the intention of interfering with his just claims to the honours of the discovery; for there is no doubt that his researches were first published to the world, and led to the actual discovery of the planet by Dr. Galle, so that the facts stated above cannot detract, in the slightest degree, from the credit due to M. Le Verrier.

6. In order not to have an inconvenient number of equations of condition, I divided the modern observations into groups, each including a period of three years, and as Mr. Airy had shown that the error of the Tabular Radius Vector was sometimes considerable, I either selected those observations which were made near opposition, or combined the others in such a manner that the result should be nearly free from the effects of this error. From the observations of each group, the error of the Tables in heliocentric longitude was found, corresponding to the time of mean opposition in the middle year of the group. Thus were formed 21 normal errors of the Tables, corresponding to as many equidistant periods between 1780 and 1840. The error for 1780 was found by interpolating between the errors of 1781, 1782, and 1783, and those given by the Ancient Observations of 1769 and 1771, and though not entitled to the same weight as the others, cannot, I think, be liable to much uncertainty. In my last calculations, I might have used more recent observations, but in order to obtain the effect due to the change of mean distance, it was necessary that the investigation should be founded on the same elements as before, and the later observations might be used as a test of the theory.

7. In order to satisfy myself that there was no important error in Bouvard's Tables, I recomputed all the principal inequalities produced by the action of Jupiter and Saturn, and found no difference of any consequence except in the equation depending on the mean longitude of Saturn minus twice that of Uranus, the error of which had been already pointed out by Bessel. The principal equation depending on the action of Jupiter, also required correction in consequence of the increased value which has been lately obtained for the mass of that planet. The corrections to be applied to Bouvard's Tables on these accounts, are the following:

$$+1''.918 \sin\{\phi_1 - 2\phi_2 - 13^\circ\ 1'.5\}$$
$$+1''.085 \sin\{\phi - \phi_2\}$$

ϕ, ϕ_1, ϕ_2 being the mean longitudes of Jupiter, Saturn, and Uranus, respectively. In the Reduction of the Greenwich Observations, the latter correction was already taken into account. M. Hansen having also found some new inequalities in the motion of Uranus, depending on the square of the disturbing force, I re-computed the values of these, following the same method as that given by M. Delaunay in the *Conn. des Temps* for 1845, and my results agreed very closely with his, the terms to be added to the longitude being

$$+32''.00 \sin\{3\phi_2 - 6\phi_1 + 2\phi + 22^\circ\ 18'.8\}$$
$$-\ 8''.35 \sin\{2\phi_2 - 6\phi_1 + 2\phi + 39\ \ 10'.5\}$$
$$-\ 1''.49 \sin\{4\phi_2 - 6\phi_1 + 2\phi + 34\ \ 48'.4\}$$

With respect to the inequalities of higher orders neglected by Bouvard, I considered that the most important of them would be, either those of long period, or those whose period was nearly equal to that of Uranus. During three-fourths of a revolution of the planet, the effects of the former class would be nearly confounded with those arising from a change in the epoch and mean motion, and those of the latter class with the effects produced by a constant change in the eccentricity and longitude of the Perihelion. The position of the planet to be determined would therefore be little affected by these terms, and the others would probably be much smaller than those which would necessarily be neglected in a first approximation to the perturbations produced by the new planet.

8. Taking into account the several corrections above-mentioned, the residual differences between the theoretical and observed heliocentric longitudes were the following:

Ancient Observations.

Year.	Observation—Theory.
1690	+61″·2
1712	+92 ·7
1715	+73 ·8
1750	−47 ·6
1753	−39 ·5
1756	−45 ·7
1764	−34 ·9
1769	−19 ·3
1771	− 2 ·3

Modern Observations.

Year.	Observation—Theory.
1780	+ 3″·46
1783	+ 8 ·45
1786	+12 ·36
1789	+19 ·02
1792	+18 ·70
1795	+21 ·38
1798	+20 ·95
1801	+22 ·21
1804	+24 ·16
1807	+22 ·07
1810	+23 ·16
1813	+22 ·00
1816	+22 ·88
1819	+20 ·69
1822	+20 ·97
1825	+18 ·16
1828	+10 ·82
1831	− 3 ·98
1834	−20 ·80
1837	−42 ·66
1840	−66 ·64

9. It is easily seen that the series expressing the correction of the *Mean* longitude in terms of the corrections applied to the elements of the orbit, is more convergent than that which gives the correction of the *true* longitude, and the same thing is true for the perturbations of the mean longitude, as compared with those of the true. The corrections found above were accordingly converted into corrections of mean longitude by multiplying each of them by the factor $\frac{r^2}{ab}$, r being the Rad. Vector, and a and b the semi-axes of the orbit. Hence these latter corrections were found to be the following:

Ancient Observations. Year.	Observation—Theory.	*Modern Observations.* Year.	Observation—Theory.
1690	+62″·6	1780	+ 3″·42
1712	+84·5	1783	+ 8·19
1715	+67·2	1786	+11·74
1750	−51·8	1789	+17·75
1753	−43·2	1792	+17·22
1756	−50·1	1795	+19·52
1764	−37·8	1798	+19·06
1769	−20·5	1801	+20·24
1771	− 2·4	1804	+22·19
		1807	+20·52
		1810	+21·89
		1813	+21·19
		1816	+22·50
		1819	+20·78
		1822	+21·50
		1825	+18·97
		1828	+11·50
		1831	− 4·29
		1834	−22·63
		1837	−46·70
		1840	−73·09

These numbers form the basis of the subsequent investigations.

10. Let $\delta\epsilon$, δa, δe, and $\delta\varpi$ denote the corrections to be applied to the Tabular Elements of Uranus, then the correction of the mean longitude at any time t is

$$= \delta\epsilon + 2e^2\,\delta\varpi + t\,\delta n - \left\{2\cos(nt+\epsilon-\varpi) + \frac{e}{2}\cos 2(nt+\epsilon-\varpi)\right\} e\,\delta\varpi$$
$$+ \left\{2\sin(nt+\epsilon-\varpi) + \frac{e}{2}\sin 2(nt+\epsilon-\varpi)\right\}\delta e$$

If we include the small term $2e^2\,\delta\varpi$ in the quantity $\delta\epsilon$, this correction may be put under the following form:

$$\delta\epsilon + t\,\delta n + \cos nt\,\delta x_1 + \sin nt\,\delta y_1 + \cos 2nt\,\delta x_2 + \sin 2nt\,\delta y_2$$

in which expression

$$\delta x_2 = \frac{1}{4} e \left\{\cos(\epsilon-\varpi)\,\delta x_1 + \sin(\epsilon-\varpi)\,\delta y_1\right\}$$
$$\delta y_2 = -\frac{1}{4} e \left\{\sin(\epsilon-\varpi)\,\delta x_1 - \cos(\epsilon-\varpi)\,\delta y_1\right\}$$