

**ELECTRIK EDUCATIONAL SERIES. ELEMENTARY
ALGEBRA. RAY'S ALGEBRA, PART FIRST: ON
THE ANALYTIC AND
INDUCTIVE METHODS OF INSTRUCTION: WITH
NUMEROUS PRACTICAL EXERCISES DESIGNED
FOR COMMON SCHOOLS AND ACADEMICS**

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JOSEPH RAY

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PREFACE.

THE object of the study of Mathematics, is two fold—the acquisition of useful knowledge, and the cultivation and discipline of the mental powers. A parent often inquires, “Why should my son study mathematics? I do not expect him to be a surveyor, an engineer, or an astronomer.” Yet, the parent is very desirous that his son should be able to reason correctly, and to exercise, in all his relations in life, the energies of a cultivated and disciplined mind. This is, indeed, of more value than the mere attainment of any branch of knowledge.

The science of Algebra, properly taught, stands among the first of those studies essential to both the great objects of education. In a course of instruction properly arranged, it naturally follows Arithmetic, and should be taught immediately after it.

In the following work, the object has been, to furnish an elementary treatise, commencing with the first principles, and leading the pupil, by gradual and easy steps, to a knowledge of the elements of the science. The design has been, to present these in a brief, clear, and scientific manner, so that the pupil should not be taught merely to perform a certain routine of exercises mechanically, but to understand the *why* and the *wherefore* of every step. For this purpose, every rule is demonstrated, and every principle analyzed, in order that the mind of the pupil may be disciplined and strengthened so as to prepare him, either for pursuing the study of Mathematics intelligently, or more successfully attending to any pursuit in life.

Some teachers may object, that this work is too simple, and too easily understood. A leading object has been, to make the pupil feel, that he is not operating on unmeaning symbols, by means of arbitrary rules; that Algebra is both a rational and a practical subject, and that he can rely upon his reasoning, and the results

of his operations, with the same confidence as in arithmetic. For this purpose, he is furnished, at almost every step, with the means of testing the accuracy of the principles on which the rules are founded, and of the results which they produce.

Throughout the work, the aim has been, to combine the clear, explanatory methods of the French mathematicians, with the practical exercises of the English and German, so that the pupil should acquire both a practical and theoretical knowledge of the subject.

While every page is the result of the author's own reflection, and the experience of many years in the school-room, it is also proper to state, that a large number of the best treatises on the same subject, both English and French, have been carefully consulted, so that the present work might embrace the modern and most approved methods of treating the various subjects presented.

With these remarks, the work is submitted to the judgment of fellow laborers in the field of education.

WOODWARD COLLEGE, August, 1848.

SUGGESTIONS TO TEACHERS.

It is intended that the pupil shall recite the Intellectual Exercises with the book open before him, as in mental Arithmetic. Advanced pupils may omit these exercises.

The following subjects may be omitted by the younger pupils, and passed over by those more advanced, until the book is reviewed.

Observations on Addition and Subtraction, Articles 50—64.

The greater part of Chapter II.

Supplement to Equations of the First Degree, Articles 164—177.

Properties of the Roots of an Equation of the Second Degree, Articles 215—217.

In reviewing the book, the pupil should demonstrate the rules on the blackboard.

The work will be found to contain a large number of examples for practice. Should any instructor deem these too numerous, a portion of them may be omitted.

To teach the subject successfully, the principles must be first clearly explained, and then the pupil exercised in the solution of appropriate examples, until they are rendered perfectly familiar.

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RAY'S
ALGEBRA.

PART FIRST.

INTELLECTUAL EXERCISES.

LESSON I.

NOTE TO TEACHERS.—All the exercises in the following lessons can be solved in the same manner as in intellectual arithmetic; yet the instructor should require the pupils to perform them after the manner here indicated. In every question let the answer be verified.

1. I have 15 cents, which I wish to divide between William and Daniel, in such a manner, that Daniel shall have twice as many as William: what number must I give to each?

If I give William a *certain number*, and Daniel twice that number, both will have 3 times that *certain number*; but both together are to have 15 cents; hence, 3 times a certain number is 15.

Now, if 3 times a certain number is 15, one-third of 15, or 5, must be the number. Hence, William received 5 cents, and Daniel twice 5, or 10 cents.

If, instead of a *certain number*, we represent the number of cents William is to receive, by x , then the number Daniel is to receive will be represented by $2x$, and what both receive will be represented by x added to $2x$, or $3x$.

If $3x$ is equal to 15,
then $1x$ or x is equal to 5.

The learner will see that the two methods of solving this question are the same in principle; but that it is more convenient to represent the quantity we wish to find, by a single letter, than by one or more words.

In the same manner, let the learner continue to use the letter x to represent the smallest of the required numbers in the following questions.

NOTE.— x is read x , or one x , and is the same as $1x$. $2x$ is read two x , or 2 times x . $3x$ is read three x , or 3 times x , and so on.

2. What number added to itself will make 12?

Let x represent the number; then x added to x makes $2x$, which is equal to 12; hence if $2x$ is equal to 12, one x , which is the half of $2x$, is equal to the half of 12, which is 6.

VERIFICATION.—6 added to 6 makes 12.

3. What number added to itself will make 16?

If x represents the number, what will represent the number added to itself? What is $2x$ equal to? If $2x$ is equal to 16, what is x equal to?

4. What number added to itself will make 24?

5. Thomas and William each have the same number of apples, and they both together have 20; how many apples has each?

6. James is as old as John, and the sum of their ages is 22 years; what is the age of each?

7. Each of two men is to receive the same sum of money for a job of work, and they both together receive 30 dollars; what is the share of each?

8. Daniel had 18 cents; after spending a part of them, he found he had as many left as he had spent; how many cents had he spent?

9. A pole 30 feet high was broken by a blast of wind; the part broken off was equal to the part left standing; what was the length of each part?

Instead of saying x added to x is equal to 30, it is more convenient to say x plus x is equal to 30. To avoid writing the word *plus*, we use the sign $+$, which means the same, and is called the sign of *addition*. Also, instead of writing the word *equal*, we use the sign $=$, which means the same, and is called the sign of *equality*.

10. John, James, and Thomas, are each to have equal shares of 12 apples; if x represents John's share, what will represent the share of James? What will represent the share of Thomas? What expression will represent $x+x+x$ more briefly. If $3x=12$, what is the value of x ? Why?

11. The sum of four equal numbers is equal to 20; if x represents one of the numbers, what will represent each of the others? What will represent $x+x+x+x$, more briefly? If $4x=20$, what is x equal to? Why?

12. What is $x+x$ equal to? Ans. $2x$.

13. What is $x+x+x$ equal to?

14. What is $x+x+x+x$ equal to?