THEORY OF MAXIMA AND MINIMA, PP. 1-186

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PREFACE

Mathematicians have always been occupied with questions of maxima and minima. With Euclid one of the simplest problems of this character was: *Find the shortest line which may be drawn from a point to a line*, and in the fifth book of the conics of Apollonius of Perga occur such problems as the *determination of the shortest line which may be drawn from a point to a given conic section.*

It is thus seen that a sort of theory of maxima and minima was known long before the discovery of the differential calculus, and it may be shown that the attempts to develop this theory exercised considerable influence upon the discovery of the calculus. Fermat, for example, after making numerous restorations of two books of Apollonius, often cites this old geometer in his "*method for determining maximum and minimum,*" *1638*, a work which in some instances is so closely related to the calculus that Lagrange, Laplace, Fourier, and others wished to consider Fermat as the discoverer of the calculus. This he probably would have been had he started from a somewhat more general point of view, as in fact was done by Newton (*Opuscula Newtoni, I*, 86–88).

Maclaurin (*A Treatise of Fluxions*, Vol. I, p. 214. 1742), wrote: "There are hardly any speculations in geometry more useful or more entertaining than those which relate to maxima and minima. Amongst the various improvements that began to appear in the higher parts of geometry about a hundred years ago, Mr. de Fermat proposed a method for finding the maxima and minima. How the methods that were then invented for the mensuration of figures and drawing tangents to curves are comprehended and improved by the method of Fluxions, may be understood from what has already been demonstrated. A general way of resolving questions concerning maxima and minima is also derived from it, that is so easy and expeditious in the most common cases, and is so successful when the question is of a higher degree, when the difficulty is greater and other methods fail us, that this is justly esteemed one of the most admirable applications of Fluxions."

The theory of maxima and minima was rapidly developed along the lines of the calculus after the discovery of the latter. Mathematicians were at first satisfied with finding the necessary conditions for the solution of the problem. These conditions, however, are seldom at the same time sufficient. In order to decide this last point, the discovery of further algebraic means was necessary. Descartes had already remarked, in a letter of March 1, 1638, that Fermat's rule for finding maxima and minima was imperfect; and we shall see that many imperfections still existed for a long time after the invention of the calculus by Newton.

As introductory to a course of lectures on the calculus of variations, I have for a number of years given a brief outline of the theory of maxima and minima. This outline is founded on the lectures that were presented by the late Professor Weierstrass in the University of Berlin. It treats the ordinary cases; that is, where the functions are everywhere regular and where the forms are either definite or indefinite. It was published as a bulletin of the University of Cincinnati in 1903. At that time I expected to publish another bulletin which was to treat the more special cases; for example, where only one-sided differentiation enters, the "ambiguous case," where the form is semi-definite, etc. A treatment of these cases, the extraordinary cases, required more study than was anticipated. The bulletin has consequently been delayed so long that I have concluded to give an entirely new exposition of the whole theory.

In the preface to the German translation by Bohlmann and Schepp of Peano's *Calcolo differenziale e principii di calcolo integrale*, Professor A. Mayer writes that this book of Peano not only is a model of precise presentation and rigorous deduction, whose propitious influence has been unmistakably felt upon almost every calculus that has appeared (in Germany) since that time (1884), but by calling attention to old and deeply rooted errors it has given an impulse to new and fruitful development.

The important objection contained in this book (Nos. 133–136) showed unquestionably that the entire former theory of maxima and minima needed a thorough renovation; and in the main Peano's book is the original source of the beautiful and to a great degree fundamental works of Scheeffer, Stolz, Victor v. Dantscher, and others, who have developed new and strenuous theories for *extreme* values of functions. Speaking for the Germans, Professor A. Mayer, in the introduction to the above-mentioned book, declares that there has been a long-felt need of a work which, for the first time, not only is free from mistakes and inaccuracies that have been so long in vogue but which, besides, so incisively penetrates an important field that hitherto has been considered quite elementary.

To a considerable degree these inaccuracies are due to one of the greatest of all mathematicians, Lagrange, and they have been diffused in the French school by Bertrand, Serret, and others. We further find that these mistakes are ever being repeated by English and American authors in the numerous new works which are constantly appearing on the calculus.

It seems, therefore, very desirable in the present state of mathematical science in this country that more attention be given to the theory of maxima and minima; for it has a high interest as a topic of pure analysis and finds immediate application to almost every branch of mathematics.

I have therefore prepared the present book for students who wish to take a more extended course in the calculus as introductory to graduate work in mathematics. I do not believe in making university students study abstruse theories in foreign languages, and in this treatise it will be found that the pedagogical side of the presentation is insisted upon; for example, the Taylor development in series is given under at least half a dozen different forms.

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