

**MATHEMATICAL
MONOGRAPHS. NO. 11.
FUNCTIONS OF A COMPLEX
VARIABLE; PP. 1-96**

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Mathematical Monographs. No. 11. Functions of a Complex Variable; pp. 1-96 by Thomas S. Fiske

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THOMAS S. FISKE

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BY

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EDITORS' PREFACE.

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THE volume called Higher Mathematics, the first edition of which was published in 1896, contained eleven chapters by eleven authors, each chapter being independent of the others, but all supposing the reader to have at least a mathematical training equivalent to that given in classical and engineering colleges. The publication of that volume is now discontinued and the chapters are issued in separate form. In these reissues it will generally be found that the monographs are enlarged by additional articles or appendices which either amplify the former presentation or record recent advances. This plan of publication has been arranged in order to meet the demand of teachers and the convenience of classes, but it is also thought that it may prove advantageous to readers in special lines of mathematical literature.

It is the intention of the publishers and editors to add other monographs to the series from time to time, if the call for the same seems to warrant it. Among the topics which are under consideration are those of elliptic functions, the theory of numbers, the group theory, the calculus of variations, and non-Euclidean geometry; possibly also monographs on branches of astronomy, mechanics, and mathematical physics may be included. It is the hope of the editors that this form of publication may tend to promote mathematical study and research over a wider field than that which the former volume has occupied.

December, 1905.

AUTHOR'S PREFACE.

IN the following pages is contained a brief introductory account of some of the more fundamental portions of the theory of functions of a complex variable. The work was prepared originally as a chapter for the volume called "Higher Mathematics," published in 1896. It has been enlarged by the addition of sections on power series, algebraic functions and their integrals, functions of two or more independent variables, and differential equations. Furthermore, the section on uniform convergence has been extended, and the treatment of Weierstrass's theorem and of Mittag-Leffler's theorem has been simplified.

It is hoped that the present work will give the uninitiated some idea of the nature of one of the most important branches of modern mathematics, and will also be useful as an introduction to larger works, such as those in English by Forsyth, Whittaker, and Harkness and Morley; in French by Jordan, Picard, Goursat, and Vallée-Poussin; and in German by Burkhardt, Stolz and Gmeiner, and Osgood.

NEW YORK, August, 1906.

CONTENTS.

ART.		Page
1.	DEFINITION OF FUNCTION	1
2.	REPRESENTATION OF COMPLEX VARIABLE	2
3.	ABSOLUTE CONVERGENCE	3
4.	ELEMENTARY FUNCTIONS	4
5.	CONTINUITY OF FUNCTIONS	5
6.	GRAPHICAL REPRESENTATION OF FUNCTIONS.	7
7.	DERIVATIVES	8
8.	CONFORMAL REPRESENTATION	11
9.	EXAMPLES OF CONFORMAL REPRESENTATION	13
10.	CONFORMAL REPRESENTATION OF A SPHERE	19
11.	CONJUGATE FUNCTIONS	20
12.	APPLICATION TO FLUID MOTION	21
13.	SINGULAR POINTS	25
14.	POINT AT INFINITY	31
15.	INTEGRAL OF A FUNCTION	32
16.	REDUCTION OF COMPLEX INTEGRALS TO REAL	36
17.	CAUCHY'S THEOREM	37
18.	APPLICATION OF CAUCHY'S THEOREM	39
19.	THEOREMS ON CURVILINEAR INTEGRALS	42
20.	TAYLOR'S SERIES	44
21.	LAURENT'S SERIES	46
22.	FOURIER'S SERIES	48
23.	UNIFORM CONVERGENCE	46
24.	POWER SERIES	54
25.	UNIFORM CONVERGENCE OF POWER SERIES	56
26.	UNIFORM FUNCTIONS WITH SINGULAR POINTS	57
27.	RESIDUES	61
28.	INTEGRAL OF A UNIFORM FUNCTION	63
29.	WEIERSTRASS'S THEOREM	66
30.	MITTAG-LEFFLER'S THEOREM	71
31.	SINGULAR LINES AND REGIONS	78
32.	FUNCTIONS HAVING n VALUES	81
33.	ALGEBRAIC FUNCTIONS	83
34.	INTEGRALS OF ALGEBRAIC FUNCTIONS	85
35.	FUNCTIONS OF SEVERAL VARIABLES	89
36.	DIFFERENTIAL EQUATIONS	91
	INDEX	99

