GEOMETRICAL PROPOSITIONS DEMONSTRATED: OR, A SUPPLEMENT TO EUCLID, BEING A KEY TO THE EXERCISES APPENDED TO EUCLID'S ELEMENTS

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PREFACE.

THE Propositions appended as Exercises to my edition of EUCLID'S ELEMENTS, will be here found demonstrated in such a way that they may be quickly mastered by any one possessing moderate capacity and application. No pains have been spared in their illustration; and the student will perceive, that the more complicated Diagrams here set before him are executed with an unusual degree of neatness. The work itself has been much developed in its plan during its progress towards completion—partly owing to the very encouraging manner in which the edition of the 'Elements' has been received by the public; but still more, to the great fertility of those Geometrical Theorems which it was intended to impress on the mind of the student. In the form of COROLLARIES or of SCHOLIA will be here found several important Propositions not contained in the Appendix to the Elements. A NOTE also has been added on the tentative or experimental methods of solving some interesting Problems, which cannot be reached by strictly geometrical constructions.

If this little volume professed to be nothing more than a HELP TO TEACHERS, it would still be laying claim

to an office, on the utility of which it were needless to expatiate. The mathematical instructor, desirous of exercising his pupils' ingenuity, and of accustoming them to wield their acquired knowledge with ease, must collect for that purpose problems not included in elementary treatises. But the labour of finding the solution or demonstration of these, however light in itself, may yet be incompatible with his occupations. Much time may be lost, even by those well acquainted with Elemental Geometry, in the attempt to discover the solution of a new problem. The most acute will often have to beat about for hours, before they hit upon the exact line of argument which leads directly to the proof. Yet so great is the allurement of discovery, and so provoking is it to be baffled in such a case, that the geometrician cannot bring himself to desist from his puzzling constructions, till compelled by head-ache and the midnight clock; even then, the unconquered problem will haunt him in his dreams. To such a one, we beg to observe, that many a valuable hour may be saved, and much weariness avoided, by having recourse to this Key.

The student who believes (and many a one grows up in the opinion) that Geometry is comprised in the series of propositions which he has learned in the Elements, or who is not aware that the properties of figures are infinite, remains extremely ignorant of the nature and extent of that science. It ought to be adopted as a constant principle in education or in self-cultivation, that a sound mind should always know

the limits of its knowledge: it should be accurately acquainted with the boundaries of its own domain, and know how wide an horizon extends beyond them. Accordingly, it is of no little importance, that one studying the Elements of Geometry should be familiar with the fact, that he does not fully explore a field of science, but only learns a pathway leading to a particular eminence in it. The Supplementary Propositions here presented to him will serve to convince him how numerous are the interesting points of view lying on both sides of that narrow pathway. He thus gets a more adequate idea of the value of his acquirement: he perceives, that though trivial in itself, it leads to a commanding position, while, at the same time, his eyes are opened to the surrounding prospect.

Those who desire to learn Trigonometry, for the sake of its application in Astronomy, Geodæsy, &c., or in general, all who would ascend to the applied Mathematics, will find their labour greatly abridged by a timely acquaintance with this little volume. Many properties of figures, which here, following in natural order, are easily demonstrated, will appear extremely difficult and abstruse when met with incidentally and isolated. A good knowledge of them will prove, to the practical mathematician, a great accession of power. Even in the common business of measuring a field, the surveyor who is thoroughly versed in the properties of figures has great advantages in respect of accuracy and brevity of method. He is better able to

adapt his mode of proceeding in each case to the circumstances of the ground; he finds more means of verification, and derives so much aid from the abundance of his own resources, as to be often enabled to dispense with multiplied observations.

These Propositions might have been easily increased in number; but they seem, at present, quite sufficient for the purpose for which they are intended; and the student who makes himself master of them, will find his sagacity and power of analysis augmented to such a degree, as to render him, in a short time, independent of foreign aids.

W. D. C.

Note.—In the following pages, the references to the theorems contained in this volume, or to the SUPPLEMENTARY PROPOSITIONS, are distinguished by the letter S. prefixed, in this manner: (S. Prop.); those not so distinguished, are made to COOLEY'S edition of EUCLID'S ELEMENTS, the Appendices to which are occasionally cited; and, as a full explanation of the abbreviated notes and symbols employed is given in that work, it was thought needless to repeat it here.

GEOMETRICAL PROPOSITIONS

DEDUCED FROM

THE FIRST SIX BOOKS

OF

EUCLID'S ELEMENTS.

FIRST BOOK.

PROP. I.

The straight line (AD) which bisects the vertical angle of an isosceles triangle (BAC) bisects the base, and is perpendicular to it.

For since in the triangles BAD, CAD, the sides BA and CA are equal (Hyp.), AD is common to both, and ∠s BAD, CAD, contained by the equal sides, are also equal (Hyp.), the remaining sides or bases BD, DC of the triangles shall be equal (I. Prop. 4); consequently BC is bisected: and also, since the ∠s ADB, ADC are equal (I. Prop. 4), AD is perpendicular to BC (Def. 7).