

**ELEMENTS OF PURE  
ARITHMETIC, OR NUMERICAL  
OPERATIONS AND THEIR  
PRIMARY RELATIONSHIPS**

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Elements of pure arithmetic, or Numerical operations and their primary relationships by  
Archibald Sandeman

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**ARCHIBALD SANDEMAN**

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E L E M E N T S  
OF  
P U R E A R I T H M E T I C,  
OR  
NUMERICAL OPERATIONS  
AND  
THEIR PRIMARY RELATIONSHIPS  
VIEWED AS THEY ARE IN THEMSELVES  
WITHOUT REGARD TO NOTATION OR SYMBOLS.

BY  
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"Puck. Yet but three? Come one more;  
Two of both kinds makes up four."

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LONDON:  
LONGMAN, BROWN, GREEN, LONGMANS, AND ROBERTS.  
MANCHESTER: THOS. SOWLER AND SONS.  
1859.

181. C. 46.



## P R E F A C E .

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THE aim of this little book is to lay a sure foundation for notational and symbolic arithmetic by setting forth shortly but thoroughly the essential nature and fundamental properties of numerical operations wholly disentangled from the consideration of any particular method of denoting numbers.

The unceasing call for easy modes of calculation in the daily business of life, along with the tendency to pass over without examination things so familiar and such mere matters of course as the simpler relations of numbers, has led to the custom of looking at arithmetic only through the medium of the ordinary system of notation and hence of mixing up together undistinguishably numerical operations and notational processes, until at last the foundation of pure science whereon the whole rests has been well nigh lost sight of and arithmetic and cyphering have become in fact almost convertible terms. Although

doubtless arithmetical results of much value can be gained, at least without a sheer waste of labour, only by means of numerical notation and notational processes, yet it is precisely the nature and properties of numerical operations strictly so called out of which that very notation and those very processes spring, without which they would not be, and by understanding which consequently they can alone be understood. The readiest, if indeed not the only, way to arrive at a true knowledge of arithmetic, and much more of its mighty offshoot algebra, seems therefore to be to seek out first of all what numerical operations and their properties really are quite irrespective of any system of numerical language and notation on the bare supposition that notions are somehow formed of numbers and that names are somehow given to them.

The importance of sound principles in such a universally used instrument of education as arithmetic is so clear that it need only be hinted.



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