NOTES ON THE CONCAVE GRATING, AND ITS APPLICATION TO STELLAR PHOTOGRAPHY; A DISSERTATION, PP. 102-131

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649239054

Notes on the Concave Grating, And Its Application to Stellar Photography; a dissertation, pp. 102-131 by S. Alfred Mitchell

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

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Trieste

NOTES ON THE CONCAVE GRATING, AND ITS APPLICATION TO STELLAR PHOTOGRAPHY

BY

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A DISSERTATION SUBMITTED TO THE BOARD OF UNIVERSITY STUDIES OF THE JOHNS HOPKINS UNIVERSITY FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

JUNE 1898

CHICAGO
The University of Chicago Press
1898

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NOTES ON THE CONCAVE GRATING.

By S. A. MITCHELL.

I. FUNDAMENTAL FORMULA.

THE general theory of the concave spherical grating has been investigated by Rowland (*Phil. Mag.*, **16**, 1883; and *Amer. Jour. Sci.* (3), **26**, 1883); by Glazebrook (*Phil. Mag.*, **15**, 1883); by Mascart (*Jour. de Phys.* (2), **2**, 1883); by Baily (*Phys. Soc. Proc.*, **5**, 1883); and by Kayser (*Winkelmann, Handbuch der Physik*, p. 408).

The following treatment is one which starts from a general condition, true for every form of grating ruled with equal spaces along a chord. By introducing the condition that the form of the grating is the section of a sphere, the general equation of the concave grating is obtained.

From the general theory of gratings (Lord Rayleigh, "Wave Theory of Light," § 14, *Ency. Brit.*, p. 437; and Kayser, *Handbuch der Physik*), we know, if we have a grating ruled with equal spaces on any surface, that

$$\lambda = \frac{\omega}{N}(\sin\gamma + \sin\mu). \qquad (1)$$

where ω is the grating space, N the order of the spectrum, γ and μ the angles which the incident and diffracted light make with the normal to the surface at any point, and λ is the wave-length.

That is, in Fig. 1, if L is the radiant point, and a cone of rays from L falling on the surface of the grating QOT at P is brought to a focus at L' by any means (if the grating is plane a lens will be necessary), then λ is the wave-length of this light.

The above equation may be written:

$$(\sin\gamma + \sin\mu) = \frac{N\lambda}{\omega} \qquad (2)$$

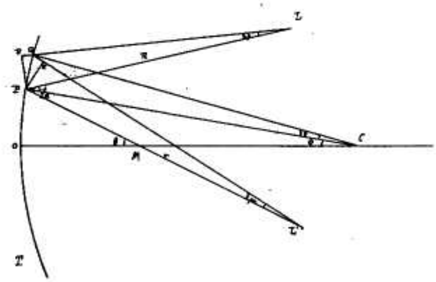

FIG. 1.

If now we allow the point P to move in the plane LoL', along the surface QOT, so that the angles which the incident and diffracted light make with the normal to the surface are now $(\gamma + \delta\gamma)$ and $(\mu + \delta\mu)$, then a new wave-length $(\lambda + \delta\lambda)$ will be brought to a focus at L', so as to satisfy the equation

$$\sin(\gamma + \delta\gamma) + \sin(\mu + \delta\mu) = \frac{N\lambda}{\omega} + \frac{\delta(N\lambda)}{\omega} \qquad (3)$$

Developing each term by Maclaurin's theorem, and subtracting (2), we get

$$\cos\gamma\, d\gamma - \tfrac{1}{2}\sin\gamma\,\delta\gamma^2 + \cos\mu\,\delta\mu - \tfrac{1}{2}\sin\mu\,\delta\mu^2 = \frac{1}{\omega}\delta(N\lambda) \qquad (4)$$

in which we neglect quantities of a higher order than the second. Hence, introducing an independent variable ϕ, we get:

$$\cos\gamma\frac{d\gamma}{d\phi} - \tfrac{1}{2}\sin\gamma\left(\frac{d\gamma}{d\phi}\right)^2 d\phi + \cos\mu\frac{d\mu}{d\phi} - \tfrac{1}{2}\sin\mu\left(\frac{d\mu}{d\phi}\right)^2 d\phi = \frac{1}{\omega}\,\frac{d(N\lambda)}{d\phi}.$$

This equation gives the change in wave-length due to a change in the angles of incidence and diffraction.

If now the surface is spherical — the case of Rowland's concave grating — O is the "center of the grating," C the center of curvature, CP is normal to the surface at P, and the line CP is the radius of curvature ρ. Then the angle LPC is equal to γ,

$L'PC$ is equal to μ, and putting OCP and OMP equal respectively to ϕ and θ, we see:

$$\gamma = \theta - \phi ; \quad \mu = \theta' - \phi ;$$

and hence,

$$\frac{d\gamma}{d\phi} = \frac{d\theta}{d\phi} - 1 ; \quad \frac{d\mu}{d\phi} = \frac{d\theta'}{d\phi} - 1 \qquad (6)$$

Calling the distances LP and $L'P$ respectively R and r, we see that by letting fall perpendiculars PR and PR' on LQ and $L'Q$ respectively that

$$PR = \rho \, d\phi \cos \gamma = R \, d\theta.$$

Hence

$$\frac{d\theta}{d\phi} = \frac{\rho \cos \gamma}{R}$$

and consequently

$$\frac{d\gamma}{d\phi} = \frac{\rho \cos \gamma}{R} - 1 \, .$$

Similarly,

$$\frac{d\mu}{d\phi} = \frac{\rho \cos \mu}{r} - 1 \, . \qquad (7)$$

Substituting these values of $\frac{d\gamma}{d\phi}$ and $\frac{d\mu}{d\phi}$ in equation (5) we get:

$$\cos \gamma \left(\frac{\rho \cos \gamma}{R} - 1 \right) + \cos \mu \left(\frac{\rho \cos \mu}{r} - 1 \right) \qquad (8)$$

$$= \frac{1}{\omega} \frac{d(N\lambda)}{d\phi} + \frac{1}{2} d\phi \left\{ \sin \gamma \left(\frac{\rho \cos \gamma}{R} - 1 \right)^2 + \sin \mu \left(\frac{\rho \cos \mu}{r} - 1 \right)^2 \right\}$$

which equation is true to terms of the order $d\phi^2$. To have a perfect focus for waves of length λ at L', this change in wave-length due to a change in the angle ϕ must be zero, or in other words $\frac{d\lambda}{d\phi} = 0$. Light of wave-lengths which are whole multiples of λ will also be brought to the same focus at L', since

$$\frac{d(2\lambda)}{d\phi} = 0, \quad \frac{d(3\lambda)}{d\phi} = 0 \quad . \quad . \quad \text{etc.}$$

Hence making $\frac{d(N\lambda)}{d\phi} = 0$, and neglecting infinitely small quantities of the first order $(d\phi)$, we get:

$$\cos \gamma \left(\frac{\rho \cos \gamma}{R} - 1 \right) + \cos \mu \left(\frac{\rho \cos \mu}{r} - 1 \right) = 0 \qquad (9)$$

It may be noted that the omitted term contains as a factor:

$$\sin\gamma\left(\frac{\rho\cos\gamma}{R}-1\right)^2+\sin\mu\left(\frac{\rho\cos\mu}{r}-1\right)^2. \qquad (10)$$

Equation (9) may be put in the form

$$\frac{\cos^2\gamma}{R}+\frac{\cos^2\mu}{r}=\frac{\cos\gamma+\cos\mu}{\rho}, \qquad (11)$$

whence we get:

$$r=\frac{R\rho\cos^2\mu}{R(\cos\gamma+\cos\mu)-\rho\cos^2\gamma}. \qquad (12)$$

This is the equation of the curve on which the spectra are brought to a focus. In this, the center of the grating is the origin, the line passing through the center of curvature is the axis of reference, R and γ the coördinates of the source of light, r and μ the coördinates of the spectral line.

This is the same equation as derived by Rowland (*loc. cit.*).

Further, we see that equation (11) is satisfied by making $R=\rho\cos\gamma$, $r=\rho\cos\mu$. The same substitution makes the omitted term, viz. (10), vanish. Consequently, if the conditions $R=\rho\cos\gamma$, $r=\rho\cos\mu$ are satisfied, formula (12) is true to terms of the second order. (See Kayser, *loc. cit.*) This condition is secured automatically in "Rowland's Mounting," where $R=\rho\cos\gamma$, $\mu=0$. Therefore $r=\rho$. (See Ames, *Johns Hopkins Circulars*, May, 1889.)

This mounting consists in having slit, grating, and camera at the vertices of a right angled triangle, the camera being placed at the center of curvature of the grating.

II. ASTIGMATISM.

One of the most important properties of a concave grating is its "astigmatism," *i. e.*, the fact that a point of light as a source gives rise to a focus, not a point, but a line. The advantages arising from this fact, as pointed out by Ames, (*Johns Hopkins Circulars*, May 1889) are:

1. A narrow spark at the slit is broadened out into a wide spectrum

2. Greater accuracy in comparing metallic and solar lines.

3. No "dust lines," as they are brought to a different focus

4. A spectrum is obtained which is broad enough to stand enlarging.

Although this property has always been recognized, no formula giving the amount of the astigmatism has ever been published, at least to my knowledge. This quantity is therefore deduced in the following pages.

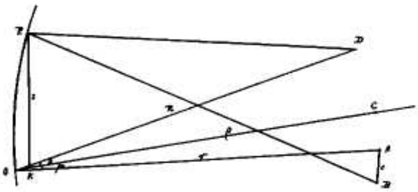

FIG. 2.

In Fig. 2 let D be the position of the point of light at the slit situated at a distance R from the center of the grating O; AB is the spectral line under consideration, which is consequently parallel to the lines of the grating, and is situated at a distance r from the center of the grating; C is the center of curvature; CO is the radius of curvature ρ. A, O, C, and D lie in a horizontal plane.

Let PO be a vertical section of the grating, *i. e.*, parallel to the lines of the grating. A small pencil of rays from the slit D falling on the grating at O is brought to a focus at A; a small pencil from D falling on the grating at P is brought to a focus at B. It is our problem to find the length of AB, assuming AB to be perpendicular to OA.

Call $AB = C$, $PK = Z$, where PK is the perpendicular let fall from P on the radius of curvature OC.

Since OK is a very small quantity compared with the other quantities considered :