APPLICATIONS OF THE CALCULUS TO MECHANICS

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PREFACE

Wherever the teaching of mathematics to engineering students is discussed, and frequently in cases of other classes of students, the criticism which is almost without exception the most insistent is this: that the student leaves the course without adequate ability to *apply* his mathematical knowledge.* This means that he has not the faculty of taking a problem, *giving it an analytic formulation*, and *interpreting the analytic results*. It is an open question whether it is the duty of the teacher of mathematics, or of the teacher of the more technical work which involves mathematics, to supply the needed training, but usually the mathematician is glad at least to share the responsibility and to do whatever he can to make his work fruitful, fully conscious of the fact that if he can successfully make the contact of his subject with the problems of the laboratory, of the engineering office, and of other activities, he will thereby add immensely to the vitality and interest of his work. With such a motive, it has been the practice at the University of Missouri to follow the course in sophomore calculus with several weeks in applications to mechanics, this being a subject rich in the kind of material desired. The present book is a formulation of the work there attempted, and it is believed that the need at our institution which has called the book into being will make its appearance welcome to a large number of mathematical departments.

Opinions will differ as to the subject-matter which such a book should contain. The authors were guided by the feeling that it was practice in applying calculus rather than a broad knowledge

* See, for instance, the reports of the joint meeting of mathematicians and engineers held in Chicago, December, 1907, under the auspices of the Chicago Section of the American Mathematical Society. These reports appeared in *Science* during the ensuing year.

of mechanics that was desired, and that such an end would be hindered rather than helped by a wide diversity of subject-matter. It was felt that, when feasible, the student secures a better insight into a subject by developing a portion of the theory himself, and so "exercises" have been introduced which form a part of such development of the theory. The "problems" are the applications of the theory, usually to cases in which numerical data are given. They vary considerably in difficulty, but it is thought that they are sufficiently numerous to supply a good number of the proper grade for any given class; and, furthermore, it is believed that a more difficult problem with a proper amount of elucidation in advance by the teacher will be of far more value to the student than a number of problems so well within his range of ability as to require very little study on his part concerning the method of attack. Some hints will be found in the text, and some suggestions are given among the answers at the end of the book.

We have endeavored, by a judicious selection of the problems to which answers are given, to secure a safe mean between the evil of supplying detailed answers, which rob the student of his independence, and the evil of furnishing him with no check upon his work. We shall be most grateful for any corrections or suggestions concerning this or other aspects of the book.

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APPLICATIONS OF THE CALCULUS TO MECHANICS

CHAPTER I

INTRODUCTION

1. Mechanics. Mechanics deals with the position or motion of bodies in space; so that the ideas of *space* and *time* implied in motion, and of *mass* implied in body, are fundamental. These three concepts cannot be defined, for there is nothing simpler in terms of which we can define them.

2. Units. With each of the three concepts mentioned is associated the idea of quantity, the quantity being measured by comparison with a conventional unit or standard quantity. In Great Britain, and for commercial purposes in the United States, the units of space, mass, and time generally employed are the foot, pound, and second respectively. The initials of these fundamental units are usually used to designate the system, "the F.P.S. system." In France, and for scientific purposes in the United States, the units are the centimeter, gram, and second respectively, giving "the C.G.S. system." The following table gives the important derived units of velocity, acceleration, and force. In engineering circles other units of mass are in use, with the result that the units of force are also changed. The "engineering" or "technical" or "gravitational" units of mass and force are also given below.*

* For further information concerning units, see Ziwet, *Theoretical Mechanics*; Encyclopedia Britannica on "Weights and Measures." In the following we shall meet with other derived units, but these will be defined as they arise.

QUANTITY	BRITISH OR F.P.S. SYSTEM	FRENCH OR C.G.S. SYSTEM
Velocity = time-rate of change of space . .	One foot per second.	One centimeter per second.
Acceleration = time-rate of change of velocity . . .	One foot per second per second. (The acceleration of a falling body near the earth's surface is $g = 32.2$ of these units, nearly.)	One centimeter per second per second. (The acceleration of a falling body near the earth's surface is $g = 981$ of these units, nearly.)
Force = Mass × Acceleration	One poundal: the force which, acting constantly throughout a second, will give to a pound of matter a unit acceleration, that is, increase the velocity by one foot per second.	One dyne: the force which, acting constantly throughout a second, will give to a gram of matter a unit acceleration, that is, increase the velocity by one centimeter per second.

The engineering units of mass and force in the two systems follow:

QUANTITY	BRITISH OR F.P.S. SYSTEM	FRENCH OR C.G.S. SYSTEM
Mass	The mass of a body weighing g pounds. (To determine the mass of a body in technical units, divide its weight in pounds by $g = 32.2$.)	The mass of a body weighing g kilograms. (To determine the mass of a body in technical units, divide its weight in kilograms by $g = 981$.)
Force = Mass × Acceleration	One pound: the force which the earth exerts at its surface upon a body weighing one pound. (One pound = g poundals.)	One kilogram: the force which the earth exerts at its surface upon a body weighing one kilogram. (One kilogram = 1000 g dynes.)

One foot = 30.5 centimeters.	One centimeter = .0328 feet.
One pound = .373 kilograms.	One kilogram = 2.68 pounds.