

ELEMENTS OF ANALYTIC GEOMETRY

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Elements of Analytic Geometry by Arthur Sherburne Hardy

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ARTHUR SHERBURNE HARDY

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ANALYTIC
GEOMETRY**

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ANALYTIC GEOMETRY.

BY
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PREFACE.

ALTHOUGH writing a text-book for the use of beginners following a short course, the tendency of an author is to sacrifice the practical value of the treatise to completeness, generalization, and scientific presentation. I have endeavored to avoid this error, which renders many works unsuitable for the class-room, however valuable they may be for reference, and yet to encourage the habit of generalization. To this end I have attempted to shun the difficulties involved in introducing the beginner to the conics, before he is familiar with their forms, through the discussion of the general equation; and at the same time to secure to him the advantages of a general analysis of the equation of the second degree. The teacher will observe the same effort to cultivate the power of general reasoning, which it is one of the objects of Analytic Geometry to promote, in the preliminary construction of loci, a process too often left in the form of a merely mechanical construction of points by substitution in the equation. In passing from Geometry to Analytic Geometry, the student should see that, while the field of operations is extended, the subject matter is essentially the same; and that what is fundamentally new is the *method*, the lines and surfaces of Geometry being replaced by their equations. His chief difficulties are:

First. A thorough understanding of the device by which this substitution is effected; hence considerable attention has been paid to this simple matter.

Second. The acquisition of an independent use of the new method as an instrument of research; hence the insertion of problems illustrative of the analytic, as distinguished from the geometric, method of proof. The function of numerical examples—that is, of examples consisting of a mere substitution of numerical values for the general constants—is simply that of testing the student's knowledge of the nomenclature. The real example in Analytic Geometry is the application of the method to the discovery of geometrical properties and forms.

The polar system has been freely used. It is not briefly explained and subsequently abandoned without application; nor is it applied redundantly to what has been already treated by the rectilinear system. It is used as one of two methods, each of which has its advantages, the selection of one or the other in any given case being governed by its adaptability to the demonstration or problem in hand.

The time allotted to the courses in Analytic Geometry for which it is hoped this treatise will be found adapted has determined the exclusion of certain topics, and has limited the chapters on Solid Geometry to the elements necessary to the student in the subsequent study of Analytic Mechanics.

ARTHUR SHERBURNE HARDY.

HANOVER, N.H., Oct. 6, 1888.

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