

**ELEMENTS OF THE
DIFFERENTIAL AND
INTEGRAL CALCULUS
WITH APPLICATIONS**

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Elements of the differential and integral calculus with applications by William S. Hall

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WILLIAM S. HALL

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WITH APPLICATIONS**

ELEMENTS
OF THE
UNIV. OF
CALIFORNIA
DIFFERENTIAL AND INTEGRAL
CALCULUS

WITH APPLICATIONS

BY

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In Memoriam
Edward Bright

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PREFACE.

THIS work is an introduction to the study of the Differential and Integral Calculus, and is intended for colleges and technical schools. The object has been to present the Calculus and some of its important applications simply and concisely, and yet to give as much as it is necessary to know in order to enter upon the study of those subjects which presume a knowledge of the Calculus. The book will be found to be adapted to the needs of the mathematical student, and also will enable the engineer to get that knowledge of the Calculus which is required by him in order to make practical applications of the subject.

All of the formulas for differentiation are established by the method of limits. This method is preferred because it is more readily understood, and is more rigorous than the method of infinitesimals; and, moreover, it has the great advantage of being a familiar method, as the student has previously used it in Algebra and Geometry. But the differential notation is fully explained, and is employed when there is any advantage gained by so doing, particularly in the investigations of the Integral Calculus.

As soon as the fundamental formulas of differentiation have been established, the corresponding inverse operations or integrations follow. Thus the essential unity of the two branches of the Calculus is emphasized, the whole subject is made more intelligible, and there is a saving of much space.

Principal applications of the Calculus, as in Maxima and Minima,

Radius of Curvature, etc., are treated at some length, while less important subjects are treated much more briefly.

A large number of carefully selected examples, some original ones, and numerous practical numerical problems from mechanics and different branches of applied mathematics are given.

As there has been an increasing demand for a short course in Differential Equations, a chapter on this subject is given which it is hoped will meet a much-felt want.

A table of Integrals, arranged for convenience of reference, is appended.

Many American and English books, and some of the leading French and German works, have been freely consulted, and problems have been gathered from many different sources.

WILLIAM S. HALL.

EASTON, PA., January, 1897.

CONTENTS.

CHAPTER I

DEFINITIONS AND FIRST PRINCIPLES.

ART.	PAGE
1. Constants and Variables	1
2. Functions	1
3. Increments	3
4. Limits	4
5. Theory of Limits	5
6. Limiting Ratio of Increments	6
7. Derivatives	8
8. Differentiation and the Differential Calculus	10

CHAPTER II

DIFFERENTIATION OF ALGEBRAIC FUNCTIONS.

9. Definition	11
10. Algebraic Sum of Functions	11
11. Product of a Constant and a Function	12
12. Any Constant	12
13. Product of Two Functions	13
14. Product of Three or More Functions	13
15. Quotient of Two Functions	14
16. Constant Power of a Function. Problems	15

CHAPTER III

DIFFERENTIATION OF TRANSCENDENTAL FUNCTIONS.

17. Definitions	19
18. Base of the Natural System of Logarithms	20
19. Logarithmic Functions	20

ART.		PAGE
20.	Exponential Function with Constant Base	21
21.	Exponential Function with Variable Base. Problems	22
22.	Circular Measure	24
23.	Limiting Value of $\frac{\sin \theta}{\theta}$	25
24.	Trigonometric Functions	26
25.	Inverse Trigonometric Functions. Problems	29

CHAPTER IV.

DIFFERENTIALS.

26.	Definition	34
27.	Geometric Interpretation of $\frac{dy}{dx}$	35
28.	Geometric Derivation of Formulas. Problems	36

CHAPTER V.

INTEGRATION.

29.	Definition	40
30.	Fundamental Formulas	40
31.	Elementary Rules	42
32.	Constant of Integration. Problems	43
33.	Integration of Trigonometric Differentials. Problems	48
34.	Definite Integrals	49
35.	Geometric Illustration of Definite Integration	50
36.	Change of Limits. Problems	52

CHAPTER VI.

SUCCESSIVE DIFFERENTIATION AND INTEGRATION.

37.	Successive Derivatives	55
38.	Successive Integration. Problems	56

APPLICATIONS IN MECHANICS.

39.	Velocity and Acceleration of Motion	59
40.	Uniformly Accelerated Motion. Problems	60
41.	Derivatives of the Product of Two Functions. Problems	61

CONTENTS.

vii

CHAPTER VII.

FUNCTIONS OF TWO OR MORE VARIABLES. IMPLICIT FUNCTIONS. CHANGE OF THE INDEPENDENT VARIABLE.

ART.		PAGE
42.	Partial Differentiation	64
43.	Total Differential of a Function of Two or More Independent Variables .	65
44.	Total Derivative when $u = f(x, y, z)$, $y = \phi(x)$, and $z = \phi_1(x)$	66
45.	Successive Partial Derivatives	68
46.	If $u = f(x, y)$, to prove that $\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$	69
47.	Implicit Functions. Problems	69
48.	Integration of Functions of Two or More Variables	71
49.	Integration of Total Differentials of the First Order. Problems	72
50.	Change of the Independent Variable. Problems	73

CHAPTER VIII.

DEVELOPMENT OF FUNCTIONS.

51.	Definition	78
52.	Maclaurin's Theorem. Problems	78
53.	Taylor's Theorem	83
54.	Demonstration of Taylor's Theorem. Problem	84
55.	Rigorous Proof of Taylor's Theorem	87
56.	Remainder in Taylor's and Maclaurin's Theorems	88
57.	Taylor's Theorem for Functions of Two or More Independent Variables .	89

CHAPTER IX.

EVALUATION OF INDETERMINATE FORMS.

58.	Indeterminate Forms	91
59.	Functions that take the Form $\frac{0}{0}$. Problems	92
60.	Functions that take the Form $\frac{\infty}{\infty}$	94
61.	Functions that take the Forms $0 \times \infty$ and $\infty - \infty$	95
62.	Functions that take the Forms 0^0 , ∞^0 and $1^{\pm\infty}$. Problems	96
63.	Compound Indeterminate Forms. Problems	98

CHAPTER X.

MAXIMA AND MINIMA OF FUNCTIONS.

64.	Definitions and Geometric Illustration	99
65.	Method of Determining Maxima and Minima	100