ACADEMIC TRIGONOMETRY: PLANE AND SPHERICAL

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Academic Trigonometry: Plane and Spherical by T. M. Blakslee

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ACADEMIC

TRIGONOMETRY.

PLANE AND SPHERICAL.

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PREFACE.

THE purpose of this arrangement is to aid the memory by noting *analogies*.

$+$ and a^2 have as *spherical analogies* $\times$ and $\cos a$. Page 12 has page 13, and each "Law" has its analogy.

In Spherical Trigonometry we note the *determining groups:* side, $+$, co. function, and $\angle$, $-$, function.

It is hoped that the Introduction will fix the characteristics of Trigonometry.

This should be accompanied by practical work, and occupy at least a week.

T. M. BLAKSLEE.

DES MOINES, IA.

NOTE. It is convenient for examinations to have tables separate from formulas.

The explanation of the use of tables should be with them.

Two pages of model solutions and answers may be added. (Opinions asked on this point.)

INTRODUCTION.

DEF. Trigonometry is, etymologically, the Science of Measuring Triangles. Besides this, it now includes the Science of Angular Functions.

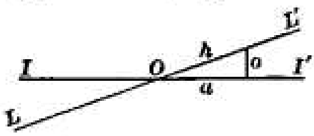
We first inquire, What is a function? then, What are the angular functions?

A function of a variable is a second variable so related to the first that any change in the variable produces a change in the function.

ILL. *Oil* in lamp and *time* it has burnt.

DEF. The functions of the angle between two straight lines are *the six ratios of the sides of the right triangle* formed by these lines and a perpendicular upon one of them from a point in the other.

NOTATION. h, hypotenuse; o, opposite; a, adjacent.



The ratios are, by definition,

$$\text{sine} = \sin = \frac{o}{h}. \quad \therefore\ o = h\sin,\ h = \frac{o}{\sin}.$$

$$\text{cosine} = \cos = \frac{a}{h}. \quad \therefore\ a = h\cos,\ h = \frac{a}{\cos}.$$

$$\text{tangent} = \tan = \frac{o}{a}. \quad \therefore\ o = a\tan,\ o = \frac{o}{\tan}.$$

And their reciprocals,

$$\frac{\sin}{\cos} = \frac{o}{h} \div \frac{a}{h} = \frac{o}{a} = \tan.$$

By similar triangles, the functions are constants for a constant angle, but variables for a variable angle.

The base line is the initial line; the hypotenuse line, the terminus of the angle.

Linear Representation. (1) If $h=1$, $o=\sin$, $a=\cos$. (2) If $a=1$, $o=\tan$.

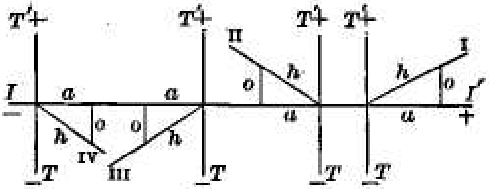
The *transverse line* is TT' through vertex and perpendicular to initial line II'.

sin = transverse projection of directed unit. (Unit h.)
cos = initial projection of directed unit.*
tan = transverse projection of h if initial projection be unity.†

Since antecedent = consequent × ratio, also
For sine and cosine, consequent = h, ratio = function.

RULE I. To obtain either side from h, multiply by ratio, sine for o, cosine for a.

RULE II. To obtain the sine from cosine, multiply by tangent.



Quadrants. II' and TT' divide the angular space about the vertex into four quadrants, numbered as in the figure.

An angle is in the quadrant in which it terminates.

* The angle being the direction of its terminus, we may speak of the ratios as *direction ratios*.

Since for the other acute angle of ratio triangle,

$$\sin=\frac{a}{h},\ \cos=\frac{o}{h},\ \text{and}\ \tan=\frac{a}{o}=\text{cotangent}=\text{cot}.$$

∴ "co" *means of complement.*

† If a circle be described with the unit base a as radius, o *is a tangent.*

The Terminal Values of the functions are as follows:

	I.	II.	III.	IV.	∠	0°.	90°.	180°.	270°.
sin	+	+	−	−	sin	0	+1	0	−1
cos	+	−	−	+	cos	+1	0	−1	0
tan	+	−	+	−	tan	0	∞	0	∞

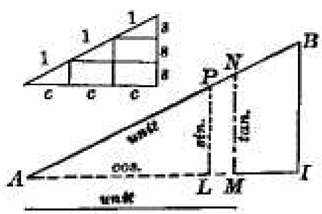
The algebraic signs being determined thus: to right and up, +; to left and down, −.

PRACTICAL DEVELOPMENT.

Wishing to calculate the distance IB to an object B, starting from I, I laid off $IA \perp IB$.

At a distance $AM = 1$ from A I erected $MN \perp AI$, determining N by looking from A to B.

I also measured AP, and drew $PL \perp AB$.

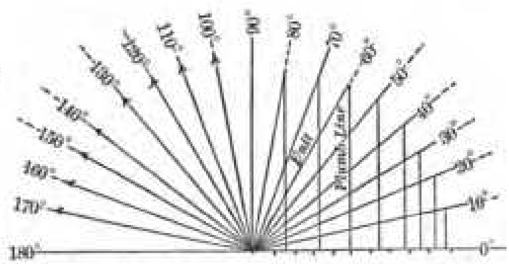


The last is not needed in measuring the distance; in fact, AM might have been any distance, when IB could have been found, as $IB = \dfrac{MN \times AI}{AM}$.

The advantage of a table of tangents is, that we never have need to construct the small triangle.

If $IA = 1000$ feet, and we have the tangent from a table, we have simply to move the decimal point three places, and we have IB at once.

Two-Place Table. Take 10 inches as an hypotenuse, and, by aid of a protractor (or by constructing an angle of 30°, geometrically, and then *trisecting it by folding*), construct the values of sine and cosine $\left(\therefore \tan = \frac{\sin}{\cos}\right)$ for every 10°. Here 10 inches = unit. $\therefore$ 0.1 inch = 0.01 unit.



Evidently (arithmetically) function $(180° - A) = f(A)$. The ratio triangles being equal, having h and A equal.

EXAMPLES.

∠°	10	20	30	40	50	60	70	80
sin	17	34	50	64	77	87	94	98
cos	98	94	87	77	64	50	34	17
tan	18	36	58	84	1.19	1.73	2.75	5.67

1. Give functions, if o, a, h, are (1) 6, 8, 10; (2) 10, 24, 26; (3) 4, 7, 5, 8.5.

2. Solve the following: ∠, o, a, h being (1) 20°, ?, ?, 100; (2) ?, 4, ?, 5; (3) 57′, 4000, ?, ?; (4) 8.8″, 4000, ?, ?.

NOTE. If the greatest angular distance of Venus from the sun be 45°, what is its distance from that body as compared to that of the earth?

3. Can the sines of 0°, 30°, 45°, 60°, and 90° be written, $\frac{1}{2}\sqrt{0}$, $\frac{1}{2}\sqrt{1}$, $\frac{1}{2}\sqrt{2}$, $\frac{1}{2}\sqrt{3}$. $\frac{1}{2}\sqrt{4}$?