

**A TREATISE ON PLANE
AND SPHERICAL
TRIGONOMETRY**

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A treatise on plane and spherical trigonometry by E. Miller

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E. MILLER

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AND SPHERICAL
TRIGONOMETRY**

~~M. H. F.~~
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A TREATISE
ON
PLANE AND SPHERICAL
TRIGONOMETRY.

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PREFACE.

THIS *Treatise on Trigonometry* has been written for use primarily in the classes of the University of Kansas.

Throughout its entire preparation constant reference has been made to the works of Serret, Lanchaupt, Young, Airy, Hind, Beasley, Todhunter, Newcomb, Chauvenet, Olney, "Oliver, Wait, and Jones," Wheeler, Peirce, Loomis, and others. The source of the material is to some extent to be found in those authors. The matter and the methods of presentation are designed to enable the student to become thoroughly acquainted with the principles and applications of Trigonometry; and care has been taken to render the demonstrations of the fundamental propositions as clear and as concise as possible, without in the least affecting their logical accuracy.

In this volume the theory of the science is based upon the analytic method, and every practical formula is illustrated by examples of numerical computation. The *Sets of Examples* given are believed to be sufficient for all practical purposes, furnishing abundance, as well as variety, of work.

The author is under obligations to Professor Webster Wells, of the Massachusetts Institute of Technology, for the Tables that appear in this volume.

Special acknowledgments are due Mr. H. B. Newson, Assistant in Mathematics, for a careful review of the manuscript, and for suggestions made.

E. MILLER,
University of Kansas.

JULY, 1894.

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TRIGONOMETRY.



CHAPTER I.

INTRODUCTION.

1. *Trigonometry* is a branch of Mathematics, and comprises all investigations relating to the numerical computation of angles and triangles.

2. *Plane Trigonometry* treats of the solution of plane triangles. It also includes the investigation of all the relations of angles, constituting the *Angular Analysis*, or *Analytical Trigonometry*.

3. *Measurement of Angles.* The *unit* of angular measurement is an angle of one degree.

A *degree* is one-ninetieth of a right angle, or of a quadrant of a circle.

Fractional parts of a degree are represented by *minutes* and *seconds*; thus,

$$\text{one minute} = \frac{1}{60} \text{ of a degree;}$$

$$\text{one second} = \frac{1}{60} \text{ of a minute.}$$

4. Symbols are used to designate degrees, minutes, and seconds. An angle or arc of 30 degrees, 35 minutes, 50 seconds, is written $30^{\circ} 35' 50''$. To compute angles or arcs by degrees, minutes, and seconds, is called the *sexagesimal* method, and is in common use.

5. The *centesimal* method consists in dividing a right angle into 100 equal parts, called *grades*; a grade into 100 equal parts, called *minutes*; and a minute into 100 equal parts, called *seconds*.

6. The ordinary conception of an angle, that it must be less than two right angles, is sufficient for geometric purposes and the solution of plane triangles and other rectilinear figures; but Trigonometry