

# **RECREATIONS IN MATHEMATICS**

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Recreations in Mathematics by H. E. Licks

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**H. E. LICKS**

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MATHEMATICS**



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By H. E. LICKS

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## PREFACE

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THE object of this book is to afford recreation for an idle hour and to excite the interest of young students in further mathematical inquiries. The topics discussed have therefore been selected with a view toward interesting students and mathematical amateurs, rather than experts and professors.

The Table of Contents is logically arranged with respect to chapters; but it will be found that within the latter, the topics are subject to no regular law or order. Some of these are long, others short; some are serious, others are frivolous; some are logical, others are absurd. It is feared that many things which might have been included have been omitted, and that still others which should have been omitted, have been included. The indulgence of readers is craved for this seeming lack of consistency and it is submitted in extenuation that the very character of the subject, partaking as it does somewhat of the nature of the curio collection, renders a more orderly treatment practically impossible.

The subject matter has been collected from many and divers sources and it is hoped that in spite of the complex nature of the work, the selection will appeal to the readers to whom it is addressed.

H. E. L.

*December, 1916.*

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# RECREATIONS IN MATHEMATICS

## CHAPTER I

### ARITHMETIC

#### 1



COUNTING a series of things and keeping tally of the tens on the fingers were processes used by primitive peoples. From the ten fingers arose ultimately the decimal system of numeration.

Recording the results of counting was done by the Egyptians and other ancient nations by means of strokes and hooks; for one thing a single stroke  $|$  was made, for two things two strokes  $||$  were used, and so on up to ten which was represented by  $\cap$ . Then eleven was written  $| \cap$ , twelve  $|| \cap$ , and so on up to twenty, or two tens, which was represented by  $\cap \cap$ . In this way the numeration proceeded up to a hundred, for which another symbol was employed.

Names for  $||$ ,  $|||$ ,  $||||$ ,  $\cap \cap$ , etc., appear in the Egyptian hieroglyphics, but a special symbol for each name is not used. Probably the Hindoos first invented such symbols, and passed them on to the Arabs, through whom they were introduced into Europe.

#### 2

### GREEK NOTATION

The Greeks used an awkward notation for recording the results of counting. The first nine letters of the Greek alphabet denoted the numbers from one to nine, so that  $\alpha$

represented one,  $\beta$  two,  $\gamma$  three, and so on. Then the following nine letters were used for ten, twenty, thirty, etc., so that  $\kappa$  represented ten,  $\lambda$  twenty,  $\mu$  thirty, and so on. Then the remaining letters  $\tau$ ,  $\upsilon$ , etc., were used for one hundred, two hundred, etc., but as the Greek alphabet had only twenty-four letters, three symbols were borrowed from other alphabets. This was an awkward notation, and there seems to have been little use made of it except to record results. A number having two letters was hence between ten and one hundred, and one having three letters was between one hundred and one thousand; thus,  $\lambda\delta$  was twenty-four and  $\tau\epsilon\delta$  was one hundred and fourteen. The Greeks were good mathematicians, as appears from their work in geometry, but only a few writers used this arithmetical numeration in computations, saying, for example, that the sum of  $\kappa\alpha$  and  $\lambda\beta$  was  $\mu\gamma$ . In those days the abacus or swan pan, similar to that seen in Chinese laundries in the United States, was employed to make arithmetical computations. From very early days this simple apparatus has been used throughout the East, and it is said that computations are made on it with great rapidity.

## 3

## ROMAN NUMERATION

The Romans represented the first five digits by I, II, III, IIII, and V, a V prefixed to the first four gave the digits from six to nine, while ten was represented by X, fifty by L and one hundred by C. This notation is still in use for a few minor purposes, it being modified by using IV for four, IX for nine, etc.; when a watch face is lettered in this notation, however, IIII is always used for IV, because Charles V said that he would allow nothing to precede a V. The

Roman notation was employed only to record numbers, and it does not appear that arithmetical operations were ever conducted with it. Perhaps this awkward notation retarded the development of mathematics among the Romans.

Frontinus, a Roman water commissioner, wrote in 97 A.D. a treatise on the Water Supply of the City of Rome, a translation of which, with an excellent commentary by Clemens Herschel, was published at Boston in 1889. A long list of the dimensions of the water pipes then in use is given, these being expressed in digits and fractions. The fraction  $1/12$  was denoted by a single horizontal stroke  $—$ ,  $2/12$  by two strokes  $==$ ,  $3/12$  by three strokes  $===$ , and so on up to  $5/12$ . Then  $1/2$  was represented by  $S$ , while the fractions from  $7/12$  to  $11/12$  were represented by adding strokes to the  $S$ , thus,  $S==$ , indicated  $1/2 + 4/12$  or  $5/6$ . The fraction  $1/24$  was indicated by  $\mathcal{L}$ . The smallest fraction used was  $1/288$  which was represented by  $\mathcal{D}$ . The following is the description of the pipe No. 50 given by Frontinus:

Fistula quinquagenaria: diametri digitos septem  $S==—\mathcal{L}\mathcal{D}$  quinque, perimetri digitos  $XXV\ \mathcal{L}\mathcal{D}\ VII$ , capit quinaris  $XLS = \mathcal{L}\mathcal{D}V$ .

Of which Herschel's translation is as follows:

The 50-pipe: seven digits, plus  $1/2$ , plus  $5/12$ , plus  $1/24$ , plus  $5/288$  in diameter; 25 digits, plus  $1/24$  plus  $7/288$  in circumference; 40 quinaris, plus  $1/2$ , plus  $2/12$ , plus  $1/24$ , plus  $5/288$  in capacity.

The digit was one-sixteenth of a Roman foot and the quantity of water flowing through a pipe of  $1\frac{1}{2}$  digits in diameter was called a quinaris. Frontinus takes the quantities of water flowing through pipes as proportional to the squares of their diameters, for he says that pipes of  $2\frac{1}{2}$  and  $3\frac{1}{2}$  digits in diameter discharge four and nine quinaris respectively.