MATHEMATICAL MONOGRAPHS. NO. 2, SYNTHETIC PROJECTIVE GEOMETRY

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GEORGE BRUCE HALSTED

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MATHEMATICAL MONOGRAPHS.

EDITED BY

MANSFIELD MERRIMAN AND ROBERT S. WOODWARD.

No. 2.

SYNTHETIC PROJECTIVE GEOMETRY.

BY

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UNDER THE TITLE

HIGHER MATHEMATICS.

First Edition, September, 1896.
Second Edition, January, 1898.
Third Edition, August, 1900.
Fourth Edition, January, 1906.

ROBERT DRUMMOND, PRINTER, NEW YORK.

EDITORS' PREFACE.

THE volume called Higher Mathematics, the first edition of which was published in 1896, contained eleven chapters by eleven authors, each chapter being independent of the others, but all supposing the reader to have at least a mathematical training equivalent to that given in classical and engineering colleges. The publication of that volume is now discontinued and the chapters are issued in separate form. In these reissues it will generally be found that the monographs are enlarged by additional articles or appendices which either amplify the former presentation or record recent advances. This plan of publication has been arranged in order to meet the demand of teachers and the convenience of classes, but it is also thought that it may prove advantageous to readers in special lines of mathematical literature.

It is the intention of the publishers and editors to add other monographs to the series from time to time, if the call for the same seems to warrant it. Among the topics which are under consideration are those of elliptic functions, the theory of numbers, the group theory, the calculus of variations, and non-Euclidean geometry; possibly also monographs on branches of astronomy, mechanics, and mathematical physics may be included. It is the hope of the editors that this form of publication may tend to promote mathematical study and research over a wider field than that which the former volume has occupied.

December, 1905.

AUTHOR'S PREFACE.

MAN, imprisoned in a little body with short-arm hands instead of wings, created for his guidance a mole geometry, a tactile space, codified by Euclid in his immortal Elements, whose basal principle is congruence, measurement.

Yet man is no mole. Infinite feelers radiate from the windows of his soul, whose wings touch the fixed stars. The angel of light in him created for the guidance of eye-life an independent system, a radiant geometry, a visual space, codified in 1847 by a new Euclid, by the Erlangen professor, Georg von Staudt, in his immortal *Geometrie der Lage* published in the quaint and ancient Nürnberg of Albrecht Dürer.

Born on the 24th of January, 1798, at Rothenburg ob der Tauber, von Staudt was an aristocrat, issue of the union of two of the few *regierenden* families of the then still free Reichsstadt, which four years later closed the 630 years of its renowned existence as an independent republic.

This creation of a geometry of position disembarrassed of all quantity, wholly non-metric, neither positively nor negatively quantitative, resting exclusively on relations of situation, takes as point of departure the since-famous quadrilateral construction. To-day it must be reckoned with from the abstractest domains of philosophy to the bread-winning marts of applied science. Thus Darboux says of it: "It seems to us that under the form first given it by von Staudt, projective geometry must become the necessary companion of descriptive geometry, that it is called to renovate this geometry in its spirit, its procedures, its applications."

KENYON COLLEGE, GAMBIER, OHIO,
December, 1905.

SYNTHETIC PROJECTIVE GEOMETRY.

INTRODUCTION.

Assumption. (*a*) The aggregate of all proper points on a straight line or 'straight' is closed or made compendent by one point at infinity or figurative point.

(*b*) With regard to a pair of different points of those on a straight all remaining fall into two classes, such that every point belongs to one and only one.

(*c*) If two points belong to different classes with regard to a pair of points, then also the latter two belong to different classes with regard to the first two. Two such point pairs are said to 'separate each other.'

(*d*) Four different points on a straight can always be partitioned in one and only one way into two pairs separating each other.

(*e*) Such separation is projective, that is, is carried on over into ejects and cuts, using the words in the sense explained in Art. 2.

Definition. (*f*) The points *A*, *B*, *C*, *D* on a straight are in the sequence *ABCD* if *AC* and *BD* are separated point pairs. Consequently this sequence is identical with the following *DABC*, *CDAB*, *BCDA*, where each letter is substituted for the one following it and the last for the first. This procedure is called cyclic permutation. Each sequence again is identical with the outcome of its own reversal, giving *DCBA*, *CBAD*, *BADC*, *ADCB*.

Theorem. (*g*) From any two such of the five sequences *ABCD*, *ABCE*, *ABDE*, *ACDE*, *BCDE*, as come from dropping each one of two consecutive elements of *ABCDE*, the other three follow.