A TREATISE ON PLANE AND SPHERICAL TRIGONOMETRY

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TREATISE

ON

PLANE AND SPHERICAL

Trigonometry.

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PREFACE

TO THE SECOND EDITION.

It is not easy to adapt a Treatise on Trigonometry to all descriptions of Students; to state, in its beginning, within a small compass, and with their simplest solutions, those Propositions which relate merely to the cases of oblique-angled triangles, and then, on the ground of those propositions and by the method of their solutions, to proceed to investigations of greater intricacy.

The Student, if he be supposed to possess a knowledge of the first six Books of Euclid, may thence, by a few easy inferences, and by the aid of some simple constructions, arrive most readily at the Trigonometrical solutions of the cases of oblique-angled triangles. If his views extend no farther, he cannot take a better guide than Ludlam or Robert Simson; nor proceed by any easier method than the *Geometrical*.

But few Students are content with such confined views. Trigonometry is now extended far beyond its original object, and to other investigations than those of the relations of the sides and angles of Triangles. The *collateral* uses of the science have become the most numerous, and are not the least important. To the knowledge of many of these, however, the Geometrical

method is unable to conduct us. At some point or other of our enquiries (we speak of its present and actual state) it must be abandoned, and recourse be had to that which technically is called the *Analytical Method.*

Since this latter is the sole thoroughly efficient method, will it not be better to make it, in a Treatise on Trigonometry, the predominant one, instead of being compelled merely to call in its aid, when the resources of the former are exhausted?

The Author of the present Treatise has endeavoured to construct it on such a plan; and, with this view, he has had as little recourse as possible, to Geometrical constructions and the properties of figures. What he thence has borrowed are not so much to be considered as the first steps in his process of demonstration, as the data and ground-work from which the process itself is to commence and to be instituted.

By these means the process is made uniform and systematical. But uniformity may be purchased at too dear a rate; and the main purpose of the Work, which is utility, would be sacrificed, if, for the sake of system, the analytical method were reluctantly compelled to submit to modes of proof that are strange to its nature and genius.

The specimens of demonstration contained in the following pages must determine whether or not such sacrifice has been made.

The great practical use of Trigonometry is the resolution of rectilinear triangles; but, that it is capable of being extended, and to objects, not merely curious, but of real interest, we may learn from the history and actual state of the science.

The first considerable extension of Trigonometry, beyond its original object, was made about twenty years after the death of

Newton. It was then, on the ground-work laid down by that great man, that three Mathematicians of the Continent, Clairaut, Dalembert and Euler, and Thomas Simpson our countryman, began to establish a system of Physical Astronomy more perfect than what its Author had left. With this view, they laid aside the Geometrical method which Newton had used, and which they thought incompetent to their purpose, and adopted the Analytical. Pursuing this method, they perceived the formulæ of Trigonometry to be of continual use and recurrence, and the language, by which the process of demonstration was conducted, to be formed, in a great degree, of symbols and phrases borrowed from that science. In order, therefore, to render the process precise and expeditious, it became necessary to improve the means and instruments by which it was carried on; and, accordingly, at the time spoken of, the advancement of Trigonometry, the pure and subsidiary science, was contemporaneous with that of Astronomy, the mixed and principal one.

This general statement would be confirmed by an examination of the Memoirs and Treatises on Physical Astronomy published about the year 1750.

Clairaut and Dalembert in their *Lunar Theories* embody in those Works, or introduce as prefatory matter, several, now commonly known, Trigonometrical formulæ *. In the Volume of Tracts which Thomas Simpson published, the Author evidently

* It will hardly be believed that theorems, such as are given in pp. 28, 29, &c. were almost unknown. Yet Clairaut, (*Mem. Acad.* 1745, p. 342, and *Theorie de la Lune*, edit. 2, p. 9.) alluding to these Theorems, says, 'M. Euler est le premier, que je sçache, qui ait fait usage de ces Theoremes pour operer sur les sinus et cosinus d'angles, sans avoir recours a leurs formes imaginaires.'

intended the one which is inserted at p. 76, as preparatory to the succeeding Theory of the Moon; and Euler distinctly states as a reason for cultivating the algorithm of sines, its great utility in the mixed Mathematics.

In the arrangement of the Treatise, which the Table of Contents sufficiently explains, Spherical succeeds to plane Trigonometry. Now, the former has not, like the latter, been extended beyond its original purpose. It has no collateral and indirect uses; it has not enriched the general language of analysis, by its peculiar phrases. But, notwithstanding this confined range, and apparent simplicity in the object of the science, its propositions are more easily established by the Analytical method than the Geometrical. And, (at least in the opinion of the Author of this Treatise) this would be the case, even if there existed no similarity and artificial connexion, between the processes by which the series of formulæ in the two branches of Trigonometry were respectively established. But, so far from there being no similarity, the corresponding propositions can be deduced by methods so analogous, that to know the one is almost to know the other.

This will appear to be the case, if we refer to pages 24 and 139, &c. of this Treatise. We shall there find similar Algebraical derivations of formulæ from two fundamental expressions for the cosine of an angle. The principle of the derivation, however, is not new; it originated with Euler, who inserted in the *Acta Acad. Petrop.* for 1779, a Memoir entitled *Trigonometria Spherica Universa, ex primis principiis breviter et dilucide derivata.* Gua next, in the *Memoirs of the Academy of Sciences* for 1783, p. 291, deduced, but by awkward and complicated processes, Spherical Trigonometry "from the Algebraical solution of the simplest of its Problems." In 1786, Cagnoli, in his excellent Treatise, derived without "similar triangles or complicated figures," the

fundamental expressions for the sine and cosine of the sum of two arcs. And lastly, Lagrange and Legendre, the one in the *Journal de L'Ecole . Polytechnique*, the other in his *Elemens de Geometrie*, have followed and simplified Euler's method, and instead of three fundamental expressions, have shewn one to be sufficient.