EASY RULES FOR THE MEASUREMENT OF EARTHWORKS BY MEANS OF THE PRISMOIDAL FORMULA

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Easy rules for the measurement of earthworks by means of the prismoidal formula by Ellwood Morris

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ELLWOOD MORRIS

EASY RULES FOR THE MEASUREMENT OF EARTHWORKS BY MEANS OF THE PRISMOIDAL FORMULA

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EASY RULES

FOR THE

MEASUREMENT OF EARTHWORKS,

BY MEANS OF THE

PRISMOIDAL FORMULA.

ILLUSTRATED WITH NUMEROUS WOODCUTS, PROBLEMS, AND EXAMPLES, AND CONCLUDED BY AN EXTENSIVE TABLE FOR FINDING THE SOLIDITY IN CUBIC YARDS FROM MEAN AREAS.

THE WHOLE

BEING ADAPTED FOR CONVENIENT USE BY ENGINEERS, SURVEYORS, CONTRACTORS, AND OTHERS NEEDING CORRECT MEASUREMENTS OF EARTHWORK.

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BY

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Dedication.

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RESPECTFULLY DEDICATED

TO THE

ENGINEERS, SURVEYORS, AND CONTRACTORS

OF

THE UNITED STATES,

BY ONE WHO IS WELL ACQUAINTED

WITH

THEIR ABILITIES AND WORTH.

TABLE OF CONTENTS,

BY CHAPTER, ARTICLE, PAGE, AND REFERENCE TO ILLUSTRATIONS.

CHAPTER I.

PRELIMINARY PROBLEMS.

		PAGE
Art. 1.	*Of the Prismoid.*—Origin probably ancient. Prismoidal Rule, first devised and demonstrated by THOMAS SIMPSON (1750). Generalized and improved by CHARLES HUTTON (1770). Now known as *the Prismoidal Formula.* A Prismoid defined, described, and illustrated by *Fig.* 1. The Prismoid, a frustum of a wedge, which latter is itself a truncated triangular Prism. Example of rectangular Prismoids, by Simpson's Rule and other processes.	7
		8
Art. 2.	*Simpson's Prismoidal Rule.* Applicable chiefly to rectangular and triangular Prismoids, with examples of each. Illustrated by *Fig.* 2.	10
		11
Art. 3.	*Hutton's Prismoidal Rules.*—His definition of a Prismoid. *General Rule,* in which the hypothetical mid-section, deduced from the ends, is first introduced by him, thus generalizing the rule, and materially extending its usefulness. Particular Rule, deduced from the wedge. Rectangular Prismoid, composed of two wedges. Initial Prismoids, to determine generality of rule. Applicability of Hutton's Rule	12
	to various solids, named by him. Example of computation by General and Particular rules; also, by Initial Prismoids. The T Prismoid, example of its computation. *Figs.* 3, 4, and 5 illustrate this article.	13 15 16
Art. 4.	*The Prismoid adapted to Earthwork.*—By Sir John Macneill, C. E. (*Fig.* 6), his figure of a prismoid composed of a prism and superposed wedge. Demonstrates the Prismoidal Rule, and uses it in an extensive series of Tables (1853).	18
Art. 5.	*The Prismoid in its Simplest Form.*—Computed by several rules. Illustrated by *Fig.* 7.	20
Art. 6.	*Further Illustration of Macneill's Prismoid.*—Double cross-sectioned in *Fig.* 8. Concise rule for earthwork wedges, confirmed by Chauvenet's Geometry. Brevity of computation of earthwork solids on level ground, by combining the rules for Wedge and Prism. Wedges applicable also to irregular ground.	21
Art. 7.	*Trapezoidal Prismoid of Earthwork, considered as two Wedges.*—Hutton's Particular Rule, modified for earthwork. Illustrated by *Fig.* 9, and by examples.	24
Art. 8.		26
	Areas of Railroad Cross-sections (within Dicdral Angles), whether Triangular, Quadrangular, or Irregular.—Triangles and Trapeziums computable by their hights and widths, taken rectangularly, however they may be placed (see *Figs.* 10, 11, 12, and 13). Exact method of equalizing irregular ground surface, by a single right line (see *Figs.* 14 and 15). Calculation of areas of cross-sections (*Figs.* 16 and 17). Very irregular cross-sections require division into elementary figures, these to be separately computed and totalized.	26 28 30 32 33 34 35
Art. 9.	*Further Illustration of the Modification of Simpson's Rule, etc.*—Example illustrated by *Fig.* 18. Diagram of Simpson's mid-section, *Fig.* 19, showing its curious composition. Diagram of the formulas of Simpson and Hutton, *Fig.* 19½. Formula deduced from it.	36 37
Art. 10.	*Adaptation of the Prismoidal Formula to Quadrature, Cubature, etc., by Simpson and Hutton.*—Explanation of it by Hutton, under the head of Equidistant Ordinates, *Fig.* 20. Example of two stations of earthwork, *Figs.* 21 to 26. Coincidence with the Prismoidal Formula. Earthwork calculations by Roots and Squares, explained and illustrated by examples, and by *Figs.* from 27 to 38; also, by comparison with other methods. Computation of the passages at grade points, from excavation to embankment. *Figs.* 39 to 42.	40 42 44 46 47 53
Art. 11.	*Cross-sections in Dicdral Angles, to find Mid-section, etc.*—Illustrated by *Figs.* 43 and 44.	55
Art. 12.	*To find Prismoidal Mean Area, from Arithmetical or Geometrical Means, from the Mid-section, or by Corrective Fractions.*—Examples, illustrated by *Figs.* 45 and 46.	59
Art. 13.	*Applicability of the Prismoidal Formula to other Solids.*—The three round bodies computable by it. Examples given and illustrated by *Fig.* 47. This formula also applies to the three square, or angular bodies.	62

		PAGE
Art. 14.	*Transformation of Solids into Equivalents more easily Computable Prismoidally.*—Equivalency explained. With examples illustrated by *Figs.* 48, 49, 50, and 51.	65
Art. 15.	*Equivalence of some important Formulas, with that for the Prismoid.*—Coincidence of the Pyramidal and Prismoidal Formulas to a certain extent, and similitude of their results within certain limits. Illustrated by *Fig.* 52 (in projection), and followed by examples calculated both Prismoidally and Pyramidally, and by other rules; all equivalent in solidity upon level ground.	68
Art. 16.	*Summary of Rules and Formulas in Chapter I.*—Indicated by numbers from I. to XII., and by reference to the various articles of the chapter in which they may be found.	74

CHAPTER II.

FIRST METHOD OF COMPUTATION, BY MID-SECTIONS DRAWN AND CALCULATED FOR AREA ON THE BASIS OF HUTTON'S GENERAL RULE.

Art. 17.	Generality and accuracy of the Prismoidal Formula. Manner of collecting the field data. Rules for cross-sectioning. Correction for curvature. Vital importance of judicious cross-sectioning. Ground to be sectioned so as to reduce it practically to plane surfaces. Warped surfaces and opposite slopes deemed inadmissible in general. Expression for the Prismoidal Formula, as generalized by HUTTON. Classification of the ground surface. Mensuration by cross-sections, long used by mathematicians, and early adopted by engineers.	79
Art. 18.	Examples in computation by our First Method. Illustrated by *Figs.* 53 to 64.	85
Art. 19.	Connected calculation of contiguous portions of Excavation and Embankment, with the passage from one to the other. Illustrated by *Figs.* 65 to 71. End sections to have, or receive, the same number of sides before calculation. Observations on mid sections. Importance of verifying all calculations. The Prismoidal Formula, the standard test for solidity. Correction for centres of gravity referred to as a refinement promoting accuracy, but not employed by engineers. This First Method easily applicable to masonry calculations. Borden's Problem, a striking example of bad cross-sectioning.	91 94 96

CHAPTER III.

SECOND METHOD OF COMPUTATION, BY HEIGHTS AND WIDTHS, AFTER SIMPSON'S ORIGINAL RULE.

Art. 20.	Prismoidal Rule, originated and demonstrated by Simpson (1750) for rectangular prismoids. Its transformation for triangular areas. Modification by direct and cross multiplication.	98
Art. 21.	Examples of computation by our Second Method. Illustrated by *Figs.* 72 to 75.	101
Art. 22.	Observations upon Simpson's Rule. Ground surfaces to be equalized to a single line. Peculiar solid where Simpson's Rule fails. See *Figs.* 81 and 82. Examples of rough ground equalized by a single line. Illustrated by *Figs.* 43 and 44. Example of a heavy embankment, from Warner's Earthwork, on uniform ground, sloping 15° transversely.	104

CHAPTER IV.

THIRD METHOD OF COMPUTATION, BY MEANS OF ROOTS AND SQUARES; A PECULIAR MODIFICATION OF THE PRISMOIDAL FORMULA, WHICH WILL BE FOUND IN PRACTICE TO BE BOTH EXPEDITIOUS AND CORRECT, IN ORDINARY CASES.

Art. 23.	Enunciation of the formula used. Comparison with the Prismoidal Formula. Use of Simpson's rule for cubature, as adopted by Hutton. Skeleton tabular arrangement of data. Tabulations for solidity. Numbered places for end-sections and mid-sections.	108
Art. 24.	Example of a heavy embankment and rock-cut (profiled in *Fig.* 76), and each computed in a body by this method. Relations of the squares of lines, or parts of lines.	110
Art. 25.	Re-computation by Roots and Squares of the examples of Chapter II. (illustrated by *Figs.* 53 to 64), showing their close agreement with this method.	114

CHAPTER V.

FOURTH METHOD OF COMPUTATION, BY REGARDING THE PRISMOID AS BEING COMPOSED OF A PRISM, WITH A WEDGE SUPERPOSED, OR OF A WEDGE AND PYRAMID COMBINED.

Art. 26.	Macneill's wedge superposed upon a prism to form a prismoid. Formula for wedge and prism to find solidity of prismoid. Discussion of the inclined wedge (*Fig.* 79), and calculation of it.	118
Art. 27.	Rules for computation by Wedge and Prism.	122
	Discussion of the wedge in connection with Chauvenet's Theorem. Illustrated by 9 small cuts. Showing the mode of computing various wedges.	123
Art. 28.	Examples of Wedge and Prism computation. Illustrated by *Figs.* 80, 14, 43, and 44.	127
Art. 29.	Peculiar solid cross-sectioned (*Fig.* 81), and in projection (*Fig.* 82). Examples of modes of computing it.	131

		PAGE
Art. 30.	The Rhomboidal Wedge and Pyramid introduced. Examples of calculation by using them to find the solidity of a prismoid.	133
	The method of using the Rhomboidal Wedge and Pyramid combined. (*Figs.* 81 and 82.)	135
	Any irregular earthwork, within certain limits, computable by Wedge and Pyramid. Process of computation. Notation of rule. The Rule itself. Its limits of range.	137
	Examples of computation by Wedge and Pyramid; direct and reverse. Re-computing by this method the examples of Chapter II., and showing their near coincidence. Illustrated by *Figs.* from 53 to 64, in Chapter II.	139
	Example of heavy embankment (from Warner's Earthwork), computed by Wedge and Pyramid.	142
	Peculiar solid again; illustrating Case 2, of the rule for Wedge and Pyramid, which correctly gives its volume.	143

CHAPTER VI.

PROFESSOR GILLESPIE'S FOUR USUAL RULES, WITH THEIR CORRECTIONS, AND A COMPARISON OF HIS CHIEF EXAMPLE WITH OUR THIRD METHOD OF COMPUTATION—BY ROOTS AND SQUARES.

Art. 31.	The late Professor Gillespie's ability and labors. The four usual rules, which he found in use. **1.** Arithmetical Average, a rule very much used, but very incorrect. The correction necessary for true results. Tabulation of an example by the Arithmetical Average, with Gillespie's correction. **2.** Middle Areas, rule explained and formula for correction to give true solidity. Tabulation of example by Middle Areas, with correction. **3.** The Prismoidal Formula, as generalized by Hutton. *The test rule for all.* Tabulation of example by it. **4.** Mean Proportionals, or Geometrical Average. Explanation of the rule. Gillespie's condemnation of Geometrical Average, and of Equivalent Level Hights, evidently hasty.	145
Art. 32.	Re-computation of example by Geometrical Average, using the grade triangle, and tabulating to intersection of side-slopes. Solidity the same as given by the Prismoidal Formula, as calculated by Gillespie himself. Generalization from which this rule flows. Equivalent Level Hights. Tabulation of an example by them to intersection of side-slopes. Resulting in solidity the same as by the Prismoidal Formula, calculated by Professor Gillespie.	151
Art. 33.	Five reliable rules for earthworks, deduced from this Chapter. Brief reference to the Core and Slope Method, one of the earliest proposed, and often re-produced, but not in general use.	152
Art. 34.	Tabulated comparison of Gillespie's example by the method of Roots and Squares, agreeing with the Prismoidal Formula, and may be used with Simpson's Multipliers. Gillespie's examples usually refer to level ground, and hence are easily computed.	154

CHAPTER VII.

PRELIMINARY OR HASTY ESTIMATES, COMPUTED BY SIMPSON'S RULE FOR CUBATURE.

Art. 35.	Nature of Preliminary Estimates. Rough and expeditious, but giving quantities proximately correct. Quantities to be always full, but never excessive. Simpson's rule for cubature. Explanation and notation. Mode of numbering the cross-sections, even and odd. Cuts and Banks to be computed separately, each in a body. General Mean Area to be found, and multiplied by length for volume.	156
Art. 36.	Simpson's Multipliers, their convenience and application. Diagrams indicating a probable construction of the rule for cubature, and its intimate connection with the Prismoidal Formula (*Figs.* 77 and 78.)	158
Art. 37.	Rough profile to be sketched, involving hights and ground slopes. Computation of a Bank by Simpson's Multipliers. Rock-cut calculated in the same way. Both illustrated by *Fig.* 76.	160
Art. 38.	Reference to the grade prism. Earth cutting, *Fig.* 29, computed by both Roots and Squares and the Hasty Process, differing only one and a half per cent.	163
	Tables (four in number) for use in Preliminary or Hasty Estimates.	165

Following this Chapter, and closing the Book, will be found an extensive TABLE OF CUBIC YARDS to mean areas for 100 feet stations (*entirely clear of error*, it is believed), giving the Cubic Yards for every foot and tenth of mean area from 0 to 1000, *by direct inspection*. And being computed accurately to three decimal places, ranges correctly up to 100,000 square feet of mean area, or to a cut 1000 feet wide, and 100 feet deep. Table preceded by explanations, and some examples of its use. This Table also operates as a general one for the conversion of *any sum* of cubic feet into Cubic Yards, by simply dividing by 100 and using the quotient *as a mean area.* 173

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CHAPTER I.

PRELIMINARY PROBLEMS.

1. *Of the Prismoid.*—Although this solid probably originated with the ancient geometers—THOMAS SIMPSON (1750), an eminent mathematician of the last century, appears to have been *the first*, in later days, to demonstrate the rule for its solidity,* now accepted by modern mensurators; and he was soon followed by Hutton, in his quarto treatise on Mensuration,† who by another process again demonstrated the Prismoidal Rule, and at the same time laid the foundations of modern mensuration, in a manner so solid, that it has come down to our time, through various editors and commentators, *substantially* (in many cases literally) *the same* as established by Hutton in his famous work of 1770.

Simpson's rule for the prismoid has been variously transformed, and written, and is now generally known by the name of *the prismoidal formula*, of which we will give hereafter the usual expressions, as well as some useful modifications, the same in substance, but often more convenient for practical purposes.

The solid called a Prismoid (from its general resemblance to a prism, and in like manner named from its base, triangular, rectangular, trapezoidal, etc.) *is a body contained between two parallel planes,*

* Simpson's Doctrine of Fluxions. (1750), 8vo, London.
† Hutton's Mensuration. (1770), 4to, Newcastle upon Tyne.